QUADRATICS UNIT

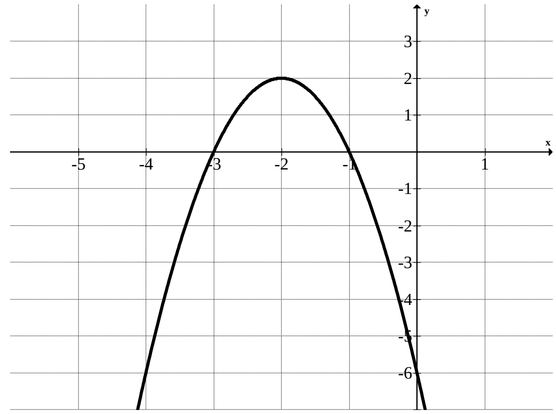
Accelerated Geom/Alg 2

MARIETTA CITY SCHOOLS

The Nature of Things

In this task you will investigate the number of real solutions to a quadratic equation.

1. $f(x) = -2x^2 - 8x - 6$



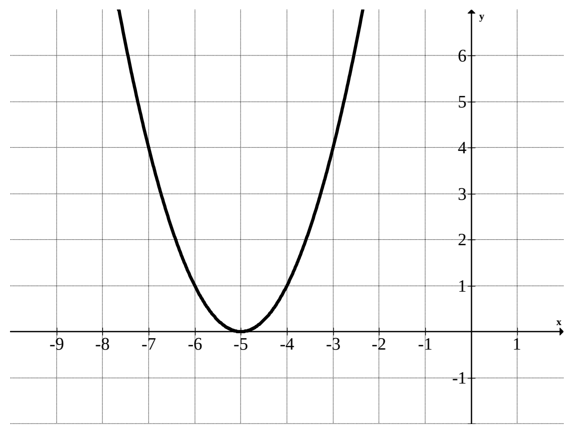
a.) How many x-intercepts are in this function? __________

b.) Label and state the x-intercept(s), if any, on the graph.

c.) Solve the quadratic function by factoring, if possible.

d) What do you notice between the x-intercepts and factored solutions?

2. $f(x) = x^2 + 10x + 25$



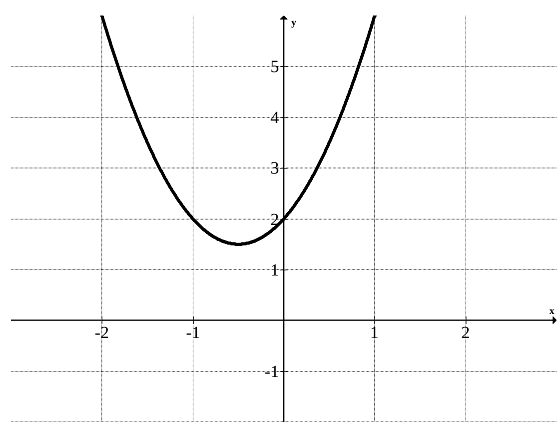
a.) How many x-intercepts are in this function? __________

b.) Label and state the x-intercept(s), if any, on the graph.

c.) Solve the quadratic function by factoring, if possible.

d) What do you notice between the x-intercepts and factored solutions?

3. $f(x) = 2x^2 + 2x + 2$



a.) How many x-intercepts are in this function? __________

b.) Label and state the x-intercept(s), if any, on the graph.

c.) Solve the quadratic function by factoring, if possible.

d) What do you notice between the x-intercepts and factored solutions?

4. Examine all previous functions and their graphs (#1-3). Look for a consistent pattern or relationship between the factored solutions and the x-intercepts on the graphs.

a.) What is true about quadratics with two real solutions?

b.) What is true about quadratics with only one real solution?

c.) What is true about quadratics with no real solutions?

Instead of observing a quadratic function's graph and/or solving it by factoring, there is an alternative way to determine its number of real solutions called the **discriminant**.

Given a quadratic function in standard form: $ax^2 + bx + c = 0, \; where \; a \neq 0$, the

Discriminant is found by using: ***$b^2 - 4ac$***

This value is used to determine the number of real solutions/zeros/roots/*x*-intercepts that exist for a quadratic equation.

Discriminant	# of real solutions
$b^2 - 4ac > 0$	____________________
$b^2 - 4ac = 0$	____________________
$b^2 - 4ac < 0$	____________________

The Imaginary Number *i*:

While solving quadratics, we have encountered situations where there have been "no *real* solutions." We have seen these both graphically and algebraically.

Why do we emphasize the word ***real***? What sets a real number apart from other number systems?

When we graph and there are no zeros/*x*-intercepts we have learned that the graph of the quadratic equation does not contain ***real*** solutions. Sketch a picture of this situation.

We have also learned that when the discriminant, ________________, is negative we will also have no ***real*** solution.

So what happens when we find ourselves in this situation? How do we handle a value that contains a negative under the radical?

By applying a new system of numbers called **Imaginary Numbers**. This new system of numbers is defined by the rule, $i = \sqrt{-1}$.

Let's see how this new rule works…

When asked to simplify $\sqrt{-24}$, we will recall: $\sqrt{24} \bullet \sqrt{-1} = \sqrt{-24}$

Since $\sqrt{-1}$ is the only real issue, we can represent this using the imaginary unit *i*.

Once you have handled the $\sqrt{-1}$, let's handle the $\sqrt{24}$ by simplifying the radical using a factor tree.

So $\sqrt{-24}$ simplifies to $2i\sqrt{6}$.

Example 1- Simplify the following radicals

a. $\sqrt{-250}$

b. $\sqrt{-81}$

c. $\sqrt{-147}$

d. $\sqrt{-17}$

So…why do we need to know how to simplify a negative radical anyway? To ***solve*** a quadratic equation!!!

Complex Numbers:

A complex number is any number that can be written with a *real* and *imaginary* term, $a+bi$, where a is the ***real*** term and bi is the ***imaginary*** term. In order to write a complex number in standard form the real term must be written first, followed by the imaginary term.

Now that we know about this new number system, we can solve any quadratic function that gives us "no *real* solution." Instead of a solution being "no *real* solutions," we are now able to respond with "two *complex* solutions." Let's try it using what we know about imaginary numbers.

$-2(x-4)^2+3=53$

Practice problems. Solve the quadratic by taking the square root. **You will no longer have "no real solution" as an answer.** We may now have two complex solutions.

1. $\left(x+6\right)^2+14 = -2$

2. $54=-2(x-4)^2$

3. $x^2+25=0$

4. $3x^2+13=-71$

Skills Practice

Solve the following quadratic equations by the best method. Use factoring, square roots, completing the square, or quadratic formula. Leave answers in simplest radical form.

1. $-6(x+3)^2 - 2 = 22$

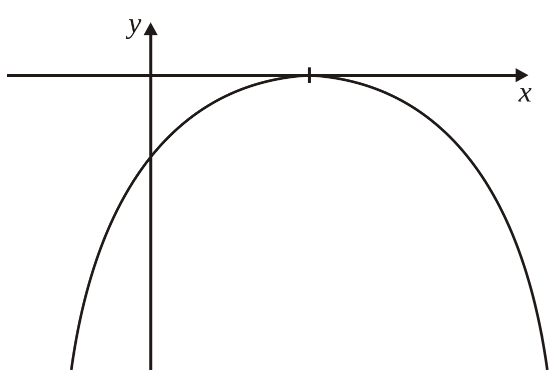
2. $-2(x-5)^2 + 4 = -18$

3. $4x^2 + 225 = 25$

4. $3(x - 1)^2 = 9$

Discriminant IB Questions

5. The diagram shows the graph of the function $y = ax^2 + bx + c$.



Complete the table below to show whether each expression is positive, negative or zero.

Expression	positive	negative	zero
a			
c			
$b^2 - 4ac$			
b			

6. Consider $f(x) = 2kx^2 - 4kx + 1$, for $k \neq 0$. The equation $f(x) = 0$ has two equal roots. Find the value of k.

7. The quadratic equation $4x^2 + 4kx + 9 = 0$, $k > 0$ has exactly one solution for x. Find the value of k.

8. The quadratic equation $kx^2 + (k - 3)x + 1 = 0$ has two equal real roots. Find the possible values of k.

Recall two other ways that you learned how to solve quadratics last year:

1.
2.

We are going to practice solving in those other manners today.

1. $2x^2 + 6x + 15 = 0$

2. $2x^2 + 6x = -7$

3. $0 = 5x^2 - 4x + 2$

4. $3x^2 + 5x - 6 = 2$

5. $x^2 - 12x + 42 = 0$

6. $2x^2 + 8x - 5 = 11$

7. $3x^2 - 18x - 5 = -32$

8. $x^2 + 24x = -152$

Solving Quadratics Practice

Solve by using the Square Root Method:

1. $4x^2 = -64$

2. $4n^2 - 7 = 317$

3. $-3x^2 = 768$

4. $8b^2 - 8 = -296$

5. $-6(x-3)^2 - 9 = -153$

6. $(y-6)^2 = -81$

7. $2(x-9)^2 = 242$

8. $-3(x+7)^2 = 27$

Solve by using the quadratic formula:

9. $3x^2 - 4x + 1 = 0$

10. $3x^2 - 2x = 5$

11. $10x^2 - 11x = -3$

12. $-2x^2 + 6 = x$

13. $2x^2 + 6x + 15 = 0$

Solve by using the completing the square:

14. $2x^2 + 32x - 72 = 0$

15. $x^2 - 6x + 52 = -4$

16. $x^2 + 12x + 105 = 5$

17. $3x^2 - 54x + 250 = 0$

Operations with Complex Numbers

Powers of *i*:
With a new system of numbers come a few more rules for simplifying.

Since we know that *i* is defined as $\sqrt{-1}$, we can use that definition to find *i* raised to any power.

$i^0 =$ ________

$i^1 =$ ________

$i^2 =$ ________

$i^3 =$ ________

We will use the pattern from above to complete the chart with the powers of *i*.

i^0	i^1	i^2	i^3	i^4	i^5	i^6	i^7	i^8	…	i^{251}
									…	

$i^{251} =$ ________

Examples – Simplify the expression.

a. i^{23} b. i^{46} c. i^{200} d. i^{157}

These new rules come in handy as we add, subtract, multiply and divide complex numbers.

Let's get started:

Addition/Subtraction: Add real parts together then add imaginary parts together.

Ex: Simplify

1) $5i+6i-4i=$ $(5+6-4)i$ $=$ $7i$

2) (1 + 2*i*) – (3 – 2*i*) =

- Careful! This is subtraction and requires an extra step that addition does not.
- Distribute neg. across parenthesis: 1 + 2*i* – 3 + 2*i*
- Collect like terms: combine real numbers 1 – 3 = -2
  combine imaginary numbers 2*i* + 2*i* = 4*i*
- Standard form: a + b*i* = -2 + 4*i*

Try these:

a) $-7i+16i-10i-2i$

b) (4 + 3*i*) + (5 + 9*i*)

c) $12i-15i+9i$

d) (2 – 9*i*) – (5 – 4*i*)

Multiplication: Use the distributive property or the area model to expand the binomials

Ex: Simplify

1) $-8i \bullet 9i^2$ $=$ $-72i^3 = -72\left(-\sqrt{-1}\right) = 72i$

2) $i\sqrt{14} \bullet i\sqrt{2}$ $= i^2\left(\sqrt{14} \bullet \sqrt{2}\right) = (-1)\left(\sqrt{28}\right) = (-1)\left(\sqrt{4}\right)\left(\sqrt{7}\right)$ $= -2\sqrt{7}$

3) $(7-6i)(-5+3i)=$

	7	-6*i*
-5	-35	30*i*
+3*i*	21*i*	$-18i^2$

$$-35+51i-18i^2 = -35+51i-18(-1)$$
$$= -35+18+51i$$
$$= -17+51i$$

Try these:

a) $i^2(2i^3)(-4i)$

b) $i\sqrt{42} \bullet i\sqrt{6}$

c) $3i(12+5i)$

d) $(4 + 2i)(-3 - 5i)$

e) $(4 - 3i)^2$

f) $(3 + 7i)(3 - 7i)$

Did you notice something special about the last problem? What is special about the product?

Special multiplication pairs, such as the final example above, have a special name: ***Conjugate Pairs***. **All** complex numbers have conjugate pairs. $(a+bi)(a-bi)$ Conjugates are used so that we can follow the important rules for radicals.

When simplifying radicals we need to be reminded of a few rules…

1. Leave No negatives under the radical
2. Leave No perfect squares under the radical
3. Leave No fractions under the radical
4. Leave No radicals in the denominator (Rationalize)

Rule #4 on the list becomes a focus of concern when we are dividing complex numbers…Because $i = \sqrt{-1}$ and the rule insists no radicals may be left in the denominator…

You must multiply by the conjugate pair, to get rid of a complex number in the denominator so the denominator becomes an integer.

Division:

1) $\dfrac{4}{2-3i}$

a) What is the conjugate of the denominator? ____________________

b) Multiply the numerator and denominator by the conjugate, then simplify

c) Express the answer in standard complex form. (a + bi) __________________________

2) $\dfrac{1+3i}{2-5i}$

a) What is the conjugate of the denominator? ____________________

b) Multiply the numerator and denominator by the conjugate, then simplify

c) Express the answer in standard complex form. (a + bi) __________________________

Try these:

a) $\frac{1-2i}{2+i}$

b) $\frac{7}{9i}$

c) $\frac{5-2i}{-3+4i}$

d) $\frac{7-4i}{9-2i}$

e) $\frac{3+2i}{i}$

f) $\frac{6+i}{7-5i}$

Simplify:

1. i^{27}
2. i^{324}
3. i^{214}
4. i^{57}
5. $16i^2 \bullet 4i^6$
6. $3i^8(-15i^3)$
7. $-5i^2(7i^3)+18i$
8. $4i(-9i)+20i^3$