DOMAIN AND RANGE OF POLYNOMIALS STATION

Another characteristic of functions that you have studied is domain and range. For each polynomial function, determine the domain and range.

Function	Domain in interval notation	Range in interval notation
g(x)	$(-\infty, \infty)$	$(-\infty, .125)$
h(x)	$(-\infty, \infty)$	$(-\infty, \infty)$
j(x)	$(-\infty, \infty)$	$(-\infty, \infty)$
k(x)	$(-\infty, \infty)$	$(-2.25, \infty)$
m(x)	$(-\infty, \infty)$	$(-\infty, \infty)$
n(x)	$(-\infty, \infty)$	$(-\infty, \infty)$

NAME AND DEGREE OF POLYNOMIAL STATION

Use the standard form equation to fill in the chart for each of the given polynomials.

Polynomial	Degree	Name by degree	No. of terms	Name by # of terms	# of Intercepts	List the Intercepts (x,y)
g(x)	2	Quadratic	2	Binomial	2	(0,0) (.5,0)
h(x)	3	Cubic	2	Binomial	3	(-1,0) (0,0) (1,0)
j(x)	3	Cubic	3	Trinomial	3	(-1,0) (0,0) (3,0)
k(x)	4	Quartic	3	Trinomial	4	(-1,0) (-2,0) (1,0) (2,0)
m(x)	5	Quintic	5	Polynomial	5	(-4,0) (-3,0) (0,0) (1,0) (2,0)
n(x)	5	Quintic	5	Polynomial	5	(-4,0) (-3,0) (0,0) (1,0) (2,0)

ZEROES OF POLYNOMIALS STATION

1. We can also describe the functions by determining some points on the functions. We can find the x-intercepts for each function as we discussed before. Under the column labeled "x-intercepts" write the ordered pairs (x,y) of each intercept and record the number of intercepts in the next column. Also record the degree of the polynomial.

Function	Degree	Number of Zeros	Zeros
g(x)	2	2	(0,0) (.5,0)
h(x)	3	3	(-1,0) (0,0) (1,0)
j(x)	3	3	(-1,0) (0,0) (3,0)
k(x)	4	4	(1,0) (-1,0) (2,0) (-2,0)
m(x)	5	5	(-4,0) (-3,0) (0,0) (1,0) (2,0)
n(x)	5	5	(-4,0) (-3,0) (0,0) (1,0) (2,0)

2. These x-intercepts are called the *zeros* of the polynomial functions. Why do you think they have this name?

$f(x) = \emptyset$

3. Make a conjecture about the relationship of degree of the polynomial and number of zeroes.

degree = # of zeros

4. *Test your conjecture* by graphing the following polynomial functions using your calculator:

Function	Degree	Zeroes	Number of Zeroes
$y = x^2$	2	(0,0)	1
$y = x^2(x-1)(x+4)$	4	(0,0) (1,0) (-4,0)	3
$y = x^2(x-1)^2$	4	(0,0) (1,0)	2

5. How are these functions different from the functions in the **first** table?

6. What is happening to the graph at some of the zeroes?

turning points

7. Now, amend your conjecture about the relationship of the degree of the polynomial and the number of x-intercepts.

degree ≥ # of zeros

8. Make a conjecture for the maximum number of x-intercepts the following polynomial function will have: $2x^7 + 4x^6 - 3x^2$

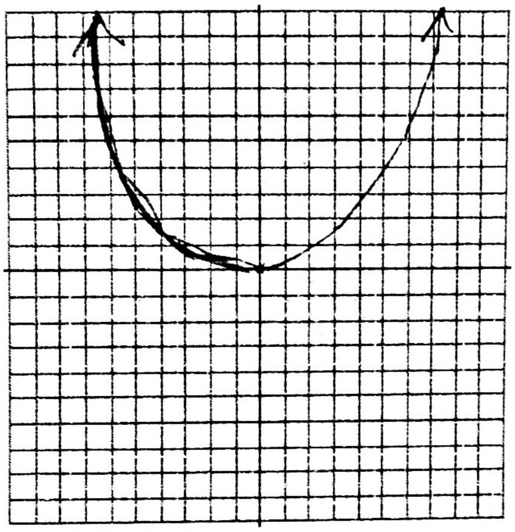
max: 7

9. Now find the x-intercepts using the calculator.

(-1.88, 0)
(-1.154, 0)
(0, 0)
(.852, 0)

SYMMETRY OF POLYNOMIALS STATION

1. Sketch a function you have seen before that has symmetry about the y-axis.

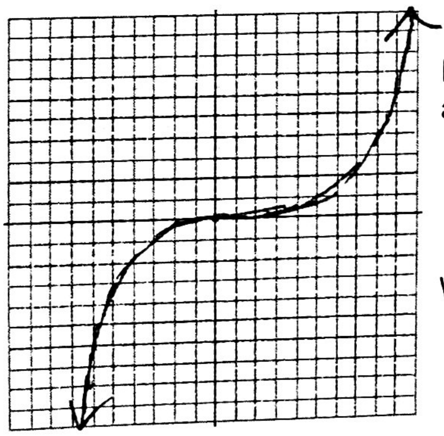


Describe in your own words what it means to have symmetry about the y-axis.

What do we call a function that has symmetry about the y-axis?

even

2. Sketch a function you have seen before that has symmetry about the origin.



Describe in your own words what it means to have symmetry about the origin.

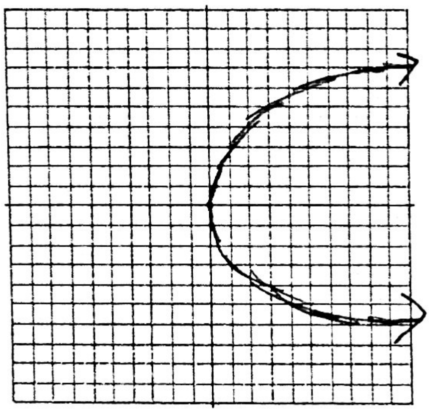
What do we call a function that has symmetry about the origin?

odd

3. Using the table below and your handout of the following six polynomial functions, classify the functions by their symmetry.

Polynomial	Symmetry about the y-axis?	Symmetry about the origin?	Even, Odd, or Neither?
g(x)			Neither
h(x)		✓	Odd
j(x)			Neither
k(x)	✓		Even
m(x)			Neither
n(x)			Neither

4. Let's talk about functions that have symmetry about the x-axis. Sketch a graph that has symmetry about the x-axis. What do you notice?



Not a function

END BEHAVIOR OF A POLYNOMIAL STATION

In determining the range of the polynomial functions, you had to consider the *end behavior* of the functions, that is the value of $f(x)$ as x approaches infinity and negative infinity.

Polynomials exhibit patterns of end behavior that are helpful in sketching polynomial functions. Using your graphs determine the following:

- is Even or Odd
- has a positive or negative leading coefficient
- rises or falls to the left
- rises or falls to the right

Complete the table by circling the appropriate characteristic for each function.

Function	Degree?	Leading Coefficient?	Left End Behavior?	Right End Behavior?
g(x)	Even Odd	Pos. Neg.	Rises Falls	Rises Falls
h(x)	Even Odd	Pos. Neg.	Rises Falls	Rises Falls
j(x)	Even Odd	Pos. Neg.	Rises Falls	Rises Falls
k(x)	Even Odd	Pos. Neg.	Rises Falls	Rises Falls
m(x)	Even Odd	Pos. Neg.	Rises Falls	Rises Falls
n(x)	Even Odd	Pos. Neg.	Rises Falls	Rises Falls

1. Write a conjecture about the **end behavior**, whether it rises or falls at the ends, of a function of the form $f(x) = ax^n$ for each pair of conditions below.

Condition a: When *n* is even and *a* > 0,

$x \to \infty, f(x) \to \infty$

$x \to -\infty, f(x) \to \infty$

Condition b: When *n* is even and *a* < 0,

$x \to \infty, f(x) \to -\infty$

$x \to -\infty, f(x) \to -\infty$

Condition c: When *n* is odd and *a* > 0,

$x \to \infty, f(x) \to \infty$

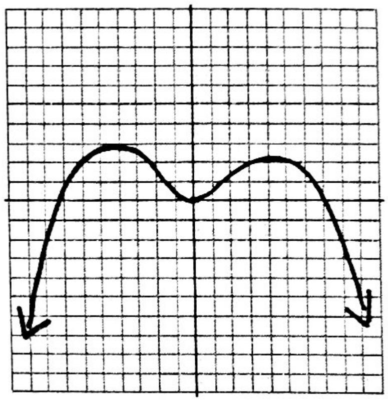
$x \to -\infty, f(x) \to -\infty$

Condition d: When *n* is odd and *a* < 0,

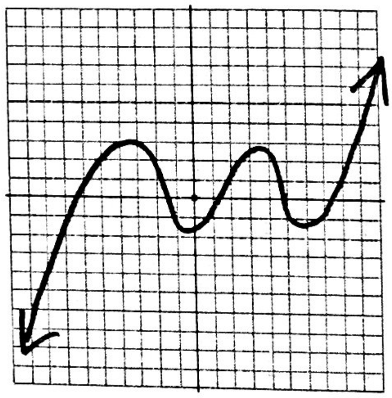
$x \to \infty, f(x) \to -\infty$

$x \to -\infty, f(x) \to \infty$

2. Based on your conjectures in part (b), sketch a fourth degree polynomial function with a negative leading coefficient.



3. Now sketch a fifth degree polynomial with a positive leading coefficient.

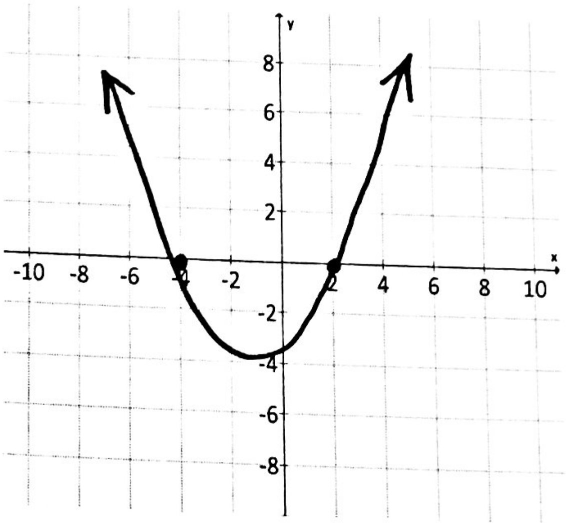


Now we can sketch the graph with the end behavior even though we cannot determine where and how the graph behaves otherwise without an equation or without the zeros.

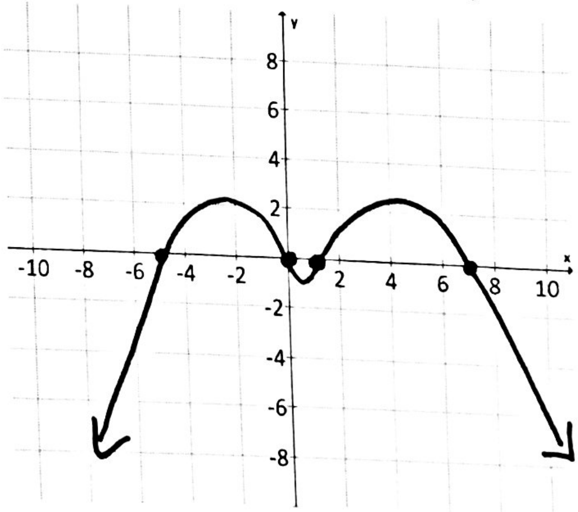
If we are given the real zeros of a polynomial function, we can combine what we know about end behavior to make a rough sketch of the function.

5. Sketch the graph of the following functions using what you know about end behavior and zeros:

a. $f(x) = (x-2)(x+4)$



b. $f(x) = -x(x-1)(x+5)(x-7)$



CRITICAL POINTS STATION

1. An **absolute maximum** and **absolute minimum** is the highest or lowest point over the entire domain of the function.

   Which of the six graphs have an absolute maximum?

   $g(x)$

   Which have an absolute minimum?

   $k(x)$

   What do you notice about the degree of these functions?

   even

2. Can you ever have an absolute maximum AND an absolute minimum in the same function? If so, sketch a graph with both. If not, why not?

   no, polynomial end behaviors either go in opposite direction (odd) or the same direction (even).

3. Other points of interest may also be where the graph begins or ends increasing or decreasing. For each of the six graphs, locate the turning points and the related intervals of increase and decrease. Then record which turning points are *relative minimum* and *relative maximum* values.

   on back

A **Relative maximum** is the highest point in a particular section of a graph of a polynomial function. A **Relative minimum** is the lowest point in a particular section of a graph of a polynomial function.

Relative maxs and mins may also be referred to as local maxs and mins.

Sometimes points that are relative minimums or maximums are also absolute minimums or absolute maximum. Are any of the relative extrema in your table also absolute extrema?

Function	Relative Minimum	Relative Maximum	Absolute Max or Min
g(x)	—	(.25, .125)	(.25, .125)
h(x)	(.577, -.385)	(-.577, .385)	—
j(x)	(-.535, -.879)	(1.869, 6.065)	—
k(x)	(-1.581, -2.25) (1.581, -2.25)	(0, 4)	(-1.581, -2.25)
m(x)	(1.619, -4.955) (-1.719, -25.395)	(-3.582, 11.144) (.482, 2.957)	—
n(x)	(-3.582, -11.144) (.482, -2.957)	(-1.719, 25.395) (1.619, 4.955)	—

5. Make a conjecture about the relationship of the degree of the polynomial and the number of turning points that the polynomial has.

Function	Degree	# of Turning Points	Intervals of Increase	Intervals of Decrease
g(x)	2	1	$(-\infty, .25)$	$(.25, \infty)$
h(x)	3	2	$(-\infty, -.577) \cap (.577, \infty)$	$(-.577, .577)$
j(x)	3	2	$(-.535, 1.869)$	$(-\infty, -.535) \cap (1.869, \infty)$
k(x)	4	3	$(-1.581, 0) \cap (1.581, \infty)$	$(0, 1.581) \cap (-\infty, -1.581)$
m(x)	5	4	$(-\infty, -3.582) \cap (-1.719, .482) \cap (1.619, \infty)$	$(-3.582, -1.719) \cap (.482, 1.619)$
n(x)	5	4	$(-3.582, -1.719) \cap (.482, 1.619)$	$(-\infty, -3.582) \cap (-1.719, .482) \cap (1.619, \infty)$