Accelerated Geom/Algebra II

Probability Unit

How Odd?

Standards Addressed
MCC9-12.S.CP.1 Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").★
MCC9-12.S.CP.7 Apply the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B), and interpret the answer in terms of the model.★

In middle school mathematics, you took a first look at probability models. You most likely solved problems that involved selecting cards, spinning a spinner, or rolling die to find the likelihood that an event occurs. In this task you will build upon what you already know. You will start with an introduction to **set theory** (a way to algebraically represent different mathematical objects). This will allow you later on in this unit to better explore two branches of probability theory: conditional probability and independence. Through these topics you will be able to uncover how data analysis and probability can help inform us about many aspects of everyday life.

Part 1 – For this task you will need a pair of six-sided dice. In Part 1, you will be concerned with the probability that one (or both) of the dice show odd values.

1. Roll your pair of dice 30 times, each time recording a success if one (or both) of the dice show an odd number and a failure if the dice do not show an odd number.

Number of Successes	Number of Failures

2. Based on your trials, what would you estimate the probability of two dice showing at least one odd number? Explain your reasoning.

3. You have just calculated an experimental probability. 30 trials is generally sufficient to estimate the theoretical probability, the probability that you expect to happen based upon fair chance. For instance, if you flip a coin ten times you expect the coin to land heads and tails five times apiece; in reality, we know this does not happen every time you flip a coin ten times.

a. A lattice diagram is useful in finding the theoretical probabilities for two dice thrown together. An incomplete lattice diagram is shown to the right. Each possible way the two dice can land, also known as an **outcome**, is represented as an ordered pair. (1,1) represents each die landing on a 1, while (4, 5) would represent the first die landing on 4 and the second one on 5.

(1, 1)	(1, 2)	(1, 3)	(,)	(,)	(,)
(2, 1)	(,)	(,)	(,)	(,)	(,)
(,)	(,)	(,)	(,)	(,)	(,)
(,)	(,)	(,)	(,)	(,)	(,)
(,)	(,)	(,)	(,)	(,)	(,)
(,)	(,)	(,)	(,)	(,)	(,)

b. Complete the lattice diagram for rolling two dice.

The 36 entries in your dice lattice represent the *sample space* for two dice thrown. The sample space for any probability model is all the possible outcomes.

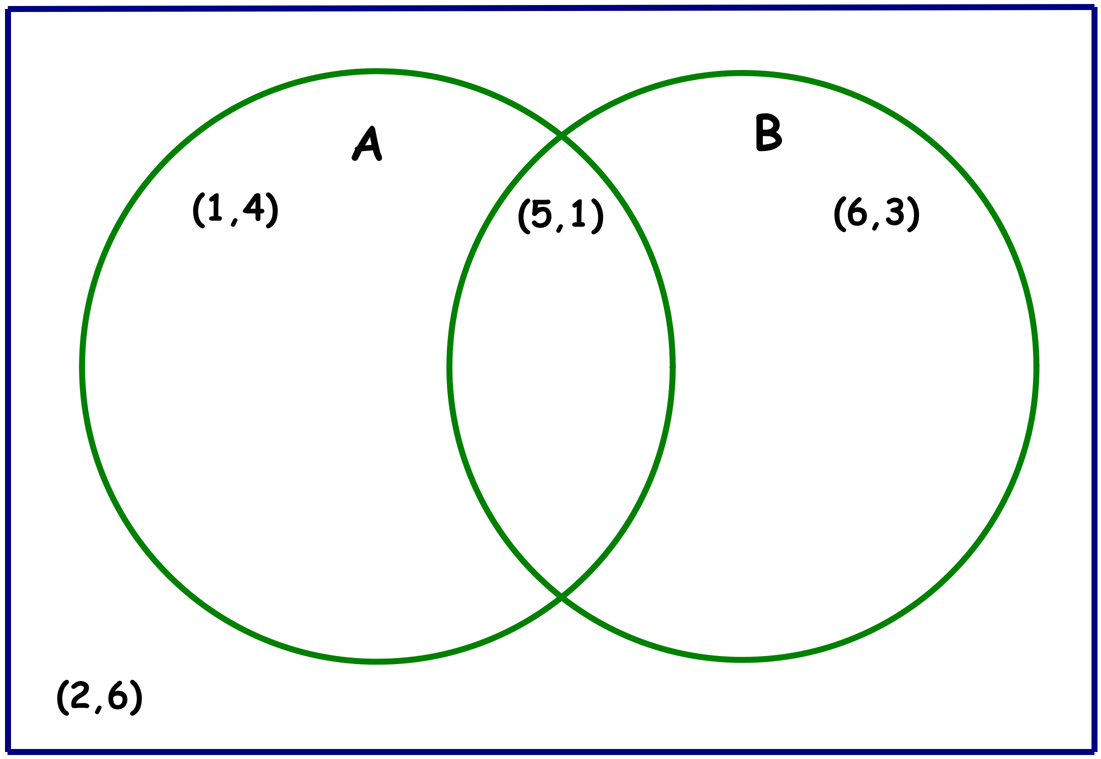
c. It is often necessary to list the sample space and/or the outcomes of a set using *set notation*. For the dice lattice above, the set of all outcomes where the first roll was a 1 can be listed as: {(1, 1), (1, 2), (1, 3), (1, 4), (1, 5), (1, 6)}. This set of outcomes is a **subset** of the set because all of the *elements* of the subset are also contained in the original set. Give the subset that contains all elements that sum to 9.

d. What is the probability that the sum of two die rolled will be 9?

e. Using your lattice, determine the probability of having at least one of the two dice show an odd number.

4. The different outcomes that determine the probability of rolling odd can be visualized using a Venn Diagram, the beginning of which is seen below. Each circle represents the possible ways that each die can land on an odd number. Circle A is for the first die landing on an odd number and circle B for the second die landing on odd. The circles overlap because some rolls of the two dice are successes for both dice. In each circle, the overlap, and the area outside the circles, one of the ordered pairs from the lattice has been placed. (1, 4) appears in circle A because the first die is odd. (6, 3) appears in circle B because the second die is odd, (5, 1) appears in both circles at the same time (the overlap) because each die is odd, and (2, 6) appears outside of the circles because neither dice is odd.

a. Finish the Venn Diagram by placing the remaining 32 ordered pairs from the dice lattice in the appropriate place.



b. How many outcomes appear in circle A? (Remember, if ordered pairs appear in the overlap, they are still within circle A).

c. How many outcomes appear in circle B?

d. The portion of the circles that overlap is called the **intersection**. The notation used for intersections is ∩. For this Venn Diagram the intersection of A and B is written $A \cap B$ and is read as "A intersect B" or "A and B." How many outcomes are in $A \cap B$?

e. When you look at different parts of a Venn Diagram together, you are considering the union of the two outcomes. The notation for unions is ∪, and for this diagram the union of A and B is written $A \cup B$ and is read "A union B" or "A or B." In the Venn Diagram you created, $A \cup B$ represents all the possible outcomes where an odd number shows. How many outcomes are in the union?

f. Record your answers to b, c, d, and e in the table below.

b. Circle A	c. Circle B	d. $\boldsymbol{A \cap B}$	e. $\boldsymbol{A \cup B}$

g. How is your answer to e related to your answers to b, c, and d?

h. Based on what you have seen, make a conjecture about the relationship of A, B, $A \cap B$ and $A \cup B$ using notation you just learned.

i. How many outcomes fall outside of $A \cup B$ (outcomes we have not yet used)? Why haven't we used these outcomes yet?

In a Venn Diagram the set of outcomes that are not included in some set is called the complement of that set. The notation used for the complement of set A is $\bar{A}$, read "A bar" or $\sim A$, read "not A." For example, in the Venn Diagram you completed above, the outcomes that are outside of $A \cup B$ are denoted $\overline{A \cup B}$.

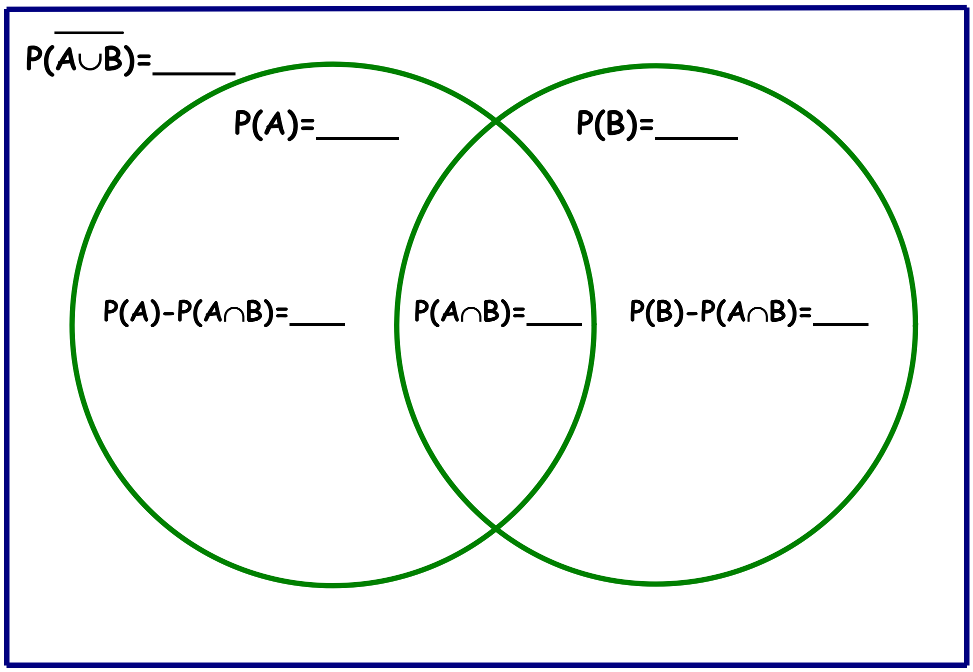
j. Which outcomes appear in $\bar{A} - B$?

k. Which outcomes appear in $\bar{B} - (\overline{A \cup B})$?

5. The investigation of the Venn Diagram in question 4 should reveal a new way to see that the probability of rolling at least one odd number on two dice is $\frac{27}{36} = \frac{3}{4}$. How does the Venn Diagram show this probability?

This probability represents $P(A \cup B)$.

6. Venn Diagrams can also be drawn using probabilities rather than outcomes. The Venn Diagram below represents the probabilities associated with throwing two dice together. In other words, we will now look at the same situation as we did before, but with a focus on probabilities instead of outcomes.



a. Fill in the remaining probabilities in the Venn Diagram.

b. Explain how you can now use the probabilities in the Venn Diagram rather than counting outcomes.

c. Use the probabilities in the Venn Diagram to find $P(\bar{B})$.

d. What relationship do you notice between $P(B)$ and $P(\bar{B})$? Will this be true for any set and its complement?

Part 2 – Venn Diagrams can also be used to organize different types of data, not just common data sets like that generated from rolling two dice. In this part of the task, you'll have an opportunity to collect data on your classmates and use a Venn Diagram to organize it.

1. Music is a popular topic amongst high school students, but one in which not all can agree upon. Let's say we want to investigate the popularity of different genres of music in your math class, particularly, Hip Hop and Country music. What genre of music do you enjoy listening to: Hip Hop, Country, or Neither?

2. Each student should identify themselves by their 3 initials (first, middle, last). **Any student who listens to both Country and Hip Hop may be listed in both categories.** Record results of the class poll in the table.

Hip Hop (HH)	Country (C)	Neither (N)

3. Draw a Venn Diagram to organize your outcomes. (Hint: Students listed in both the Hip Hop and Country categories should be identified first prior to filling in the diagram.)

4. Find $P(HH)$.

5. Find $P(\bar{C})$.

6. Find $P(HH \cap C)$.

7. Find $P(HH \cup C)$.

8. In part 1, you found the relationship between A, B, $A \cap B$ and $A \cup B$ to be $A \cup B = A + B - A \cap B$. In a similar way, write a formula for $P(A \cup B)$.

9. Now find $P(HH \cup C)$ using the formula instead of the Venn Diagram. Did you get the same answer as you did in f above?

10. In what situation might you be forced to use the formula instead of a Venn Diagram to calculate the union of two sets?

Part 3 – Now that you have had experience creating Venn Diagrams on your own and finding probabilities of events using your diagram, you are now ready for more complex Venn Diagrams.

1. In this part of the task, you will be examining data on the preference of social networking sites based on gender. Again, you will collect data on students in your class, record the data in a two-way frequency table, and then create a Venn Diagram to organize the results of the poll. Which social networking site do you prefer?

2. Record results from the class poll in the table.

	Twitter (T)	**Instagram (I)**
Female (F)		
Male (M)		

3. Draw a Venn Diagram to organize your outcomes. (Hint: Notice that male and female will not overlap and neither will Twitter and Instagram).

4. Find $P(T \cup M)$.

5. What is another way to write the probability $P(T \cup M)$ of using a complement?

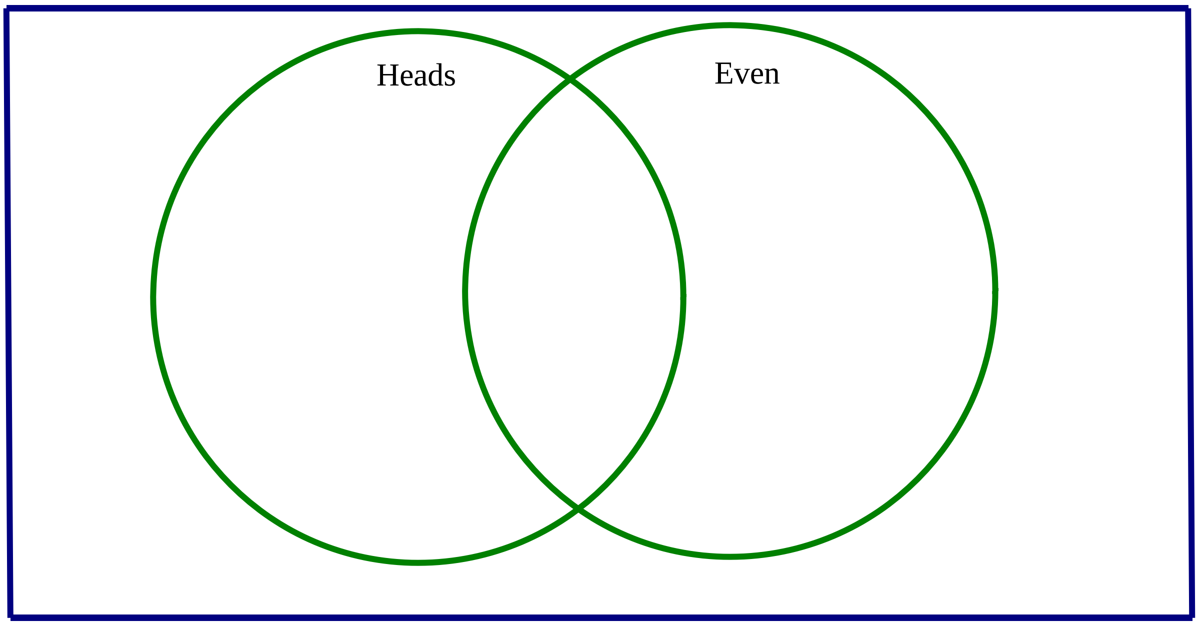
6. Find $P(\bar{I} \cap F)$.

7. Find $P(T \cap M) + P(\overline{T \cup M})$

Probability

1. Create a Venn Diagram to describe the following event:

   Rolling a die and flipping a coin (H1, T1, H2, T2, etc.)



Find the following probabilities:

a) P(H) = ____________ b) P(E) = _____________ c) P(H∩E) = _______________

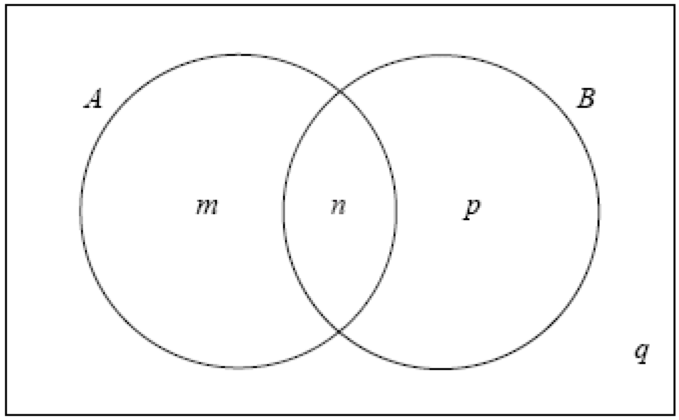
d) P(H∪E) = ______________ e) $P(\overline{H \cup E})$ = _____________

2. If events A and B are independent and P(A) = 0.15 and P(B) = 0.64, find the following:

a) $P(A \cap B)$ = ___________ b) $P(A \cup B)$ = ____________ c) $P(\bar{A})$ = ______________

d) $P(\bar{A} - \bar{B})$ = ____________

3. The Venn diagram below shows events A and B where $P(A) = 0.5$, $P(\overline{A \cup B}) = 0.3$ and $P(A \cap B) = 0.2$. The values m, n, p and q are probabilities.



(a) (i) Write down the value of n.

(ii) Find the value of m, of p, and of q.

(b) Find $P(B')$.

4. What is the probability of rolling a pair of dice and obtaining a total score of 10 or more?

5. A card is drawn at random from a deck. What is the probability that it is an ace or a king?

6. What is the probability of rolling a 5 or a 6 on a fair die?

7. What is the probability of rolling an odd?

8. What is the probability of rolling a prime?

9. What is the probability of rolling an odd or a prime?

Use the following to answer questions 10 through 12:
If you draw an M&M candy at random from a bag of M&M's, the candy you draw will have one of six colors. The probability of drawing each color depends on the proportion of each color among all candies made. Assume the table below gives the probability that a randomly chosen M&M has each color.

Color	Brown	Red	Yellow	Green	Orange	Tan
Probability	.3	.3	?	.1	.1	.1

10. The probability of drawing a yellow candy is
    A) 0.1. B) 0.2. C) 0.3. D) 3.
    E) impossible to determine from the information given.

11. The probability of not drawing a red candy is
    A) 0.1. B) 0.3. C) 0.6. D) 0.7. E) 0.9.

12. The probability of drawing neither a brown nor a green candy is
    A) 0.1. B) 0.3. C) 0.4. D) 0.6. E) 0.7.

Describe the sample space for questions 13 through 15.

13. Toss a coin four times and record the results of each of the four tosses in order. A typical outcome is then HTTH. Counting shows there are 16 possible outcomes. The sample space is the set of all 16 strings of four H's and T's.

14. Suppose our only interest is the number of heads in four tosses. Now we can be exact in a similar fashion. The random phenomenon is to toss a coin four times and count the number of heads.

15. Flip a coin and roll a die. Possible outcomes are a head (H) followed by any digit 1 to 6, or a tail (T) followed by any digit 1 to 6.

16. Janice and Lee take set shots at a netball goal from 3m. From past experience, Janice throws a goal in average 2 times in every 3 shots, where as Lee throws a goal 4 times in every 7. If they shoot for goals, determine the probability that:

a) both score a goal

b) both miss

c) Janice scores but Lee misses

17. When a nut was tossed 400 times it finished on its edge 84 times and on its side for the rest. Use this information to estimate the probability that when two identical nuts are tossed:

a) they both fall on their edges

b) the both fall on their sides

18. Suppose an equally divided spinner is spun twice. It is divided up into colors; blue, green, blue, yellow.

a) What is the probability that blue appears on both spins?

b) What is the probability that yellow appears on both spins?

c) What is the probability that different colors appear on both spins?

d) What is the probability that blue appears on either spin?

19. A box contains 6 red and 3 yellow tickets. Two tickets are drawn at random (the first being replaced before the second is drawn). Determine the probability that:

a) both are red

b) both are yellow

c) the first is red and the second is yellow

d) the first is yellow and the second is red

20. Seven tickets are numbered 1-7 and are placed in a hat. Two of the tickets are taken from the hat at random without replacement. Determine the probability that:

a) both are odd

b) both are even

c) the first is even and the second is odd

d) one is even and the other is odd

Conditional Probability

Part 1 – ICE CREAM

The retail and service industries are another aspect of modern society where probability's relevance can be seen. By studying data on their own service and their clientele, businesses can make informed decisions about how best to move forward in changing economies. Below is a table of data collected over a weekend at a local ice cream shop, Frankie's Frozen Favorites. The table compares a customer's flavor choice to their cone choice.

Frankie's Frozen Favorites	**Chocolate**	**Butter Pecan**	**Fudge Ripple**	**Cotton Candy**
Sugar Cone	36	19	34	51
Waffle Cone	35	56	35	24

1. By looking at the table, but without making any calculations, would you say that there is a relationship between flavor and cone choice? Why or why not?

2. Find the following probabilities (write as percentages):

a. $P(W)$

b. $P(S)$

c. $P(C)$

d. $P(BP)$

e. $P(FR)$

f. $P(CC)$

In order to use probability to reinforce the connection between ice cream flavors and the type of cone chosen, you will calculate **conditional probabilities**.

By considering only the people that like Fudge Ripple ice cream, what is the probability that they will choose a waffle cone?

When calculating conditional probability, it is common to use the term "given." In the above question you have calculated the probability of a choosing a waffle cone given that they liked Fudge Ripple ice cream. The condition (or, "given") is denoted with a single, vertical bar separating the probability needed from the condition. The probability of a choosing a waffle cone given that they liked Fudge Ripple ice cream is written as *P(W|FR)*.

3. In order to better investigate the correlation between flavor and cone choice, calculate the conditional probabilities for each cone given each flavor choice. A table has been provided to help organize your calculations.

Frankie's Frozen Favorites	**Chocolate**	**Butter Pecan**	**Fudge Ripple**	**Cotton Candy**
Sugar Cone	P(S|C)	P(S|BP)	P(S|FR)	P(S|CC)
Waffle Cone	P(W|C)	P(W|BP)	P(W|FR)	P(W|CC)

4. Compare and contrast the probabilities you found in question 2 with the conditional probabilities you found in question 3. Which flavors actually affect cone choice? Which do not? How did you make this determination?

5. Now calculate all eight of the probabilities with the given being the cone choice. For example, P(C|W).

6. How do the probabilities compare?

Part 2 – Lung Cancer and Smoking

Say-No-To-Smoking campaigns are vigilant in educating the public about the adverse health effects of smoking cigarettes. This motivation to educate the public has its beginnings in data analysis. Below is a table that represents a sampling of 500 people. Distinctions are made on whether or not a person is a smoker and whether or not they have ever developed lung cancer. Each number in the table represents the number of people that satisfy the conditions named in its row and column.

	Has been a smoker for	Has not been a smoker
Has not developed lung cancer	202	270
Has developed lung cancer	23	5

1. How does the table indicate that there is a connection between smoking and lung cancer?

2. Using the 500 data points from the table, you can make reasonable estimates about the population at large by using probability. 500 data values are considered, statistically, to be large enough to draw conclusions about a much larger population. In order to investigate the table using probability, use the following outcomes:

   S – The event that a person is a smoker
   L – The event that a person develops lung cancer

   Find each of these probabilities (write as percentages):

   a) $P(S)$
   b) $P(\bar{S})$
   c) $P(L)$
   d) $P(\bar{L})$
   e) $P(L \cap S)$
   f) $P(\bar{S} \cap \bar{L})$
   g) $P(S \cap \bar{L})$
   h) $P(S \cup L)$
   i) $P(\bar{S} \cup \bar{L})$
   j) $P(\bar{S} \cap L)$

There are two ways that you can compute conditional probability. You have seen one way and you are about to learn another. So far, you have seen how to compute conditional probability using a two way table. Compute the probability of $P(S|L)$.

Part 3 – A Formula for Conditional Probability

The formulaic definition of conditional probability can be seen by looking at the different probabilities you calculated in part 3. The formal definition for the probability of event A given event B is the chance of both events occurring together with respect to the chance that B occurs. As a formula,

Probability of *A* given *B*

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

In part 4 you found that $P(L) = \frac{28}{500}$ and $P(S \cap L) = \frac{23}{500}$. Using the formula for conditional probability is another way to determine that $P(S|L) = \frac{23}{28}$.

$$P(S|L) = \frac{P(S \cap L)}{P(L)} = \frac{\frac{23}{500}}{\frac{28}{500}} = \frac{23}{500} \div \frac{28}{500} = \frac{23}{500} \cdot \frac{500}{28} = \frac{23}{28}$$

1. Using the same approach that is shown above, show that the conditional probability formula works for $P(\bar{S}|\bar{L})$.

2. For two events S and Q it is known that $P(Q) = .45$ and $P(S \cap Q) = .32$. Find $P(S|Q)$.

3. For the events X and Y it is known that $P(Y) = \frac{1}{5}$ and $P(X \cap Y) = \frac{2}{15}$. Find $P(X|Y)$.

4. For two events B and C it is known that $P(C|B) = .61$ and $P(C \cap B) = .48$. Find $P(B)$.

5. For the events V and W it is known that $P(W) = \frac{2}{9}$ and $P(V|W) = \frac{2}{11}$. Find $P(V \cap W)$.

6. For two events G and H it is known that $P(H|G) = \frac{5}{14}$ and $P(H \cap G) = \frac{1}{3}$. Explain why you cannot determine the $P(H)$.

Part 4 – Test for Independence

Consider the situation that the probability of being in a world history class as a sophomore is $P(WH) = 0.90$, being in analytic geometry as a sophomore is $P(AG) = 0.65$. The probability of being in both world history and analytic geometry as a sophomore is $P(WH \cap AG) = 0.585$. Calculate the $P(AG|WH)$.

Do you see a connection between $P(AG|WH)$ and $P(AG)$? If so, what is it?

Multiple events in probability are said to be **independent** if the outcome of any one event does not affect the outcome of the others.

In this case, it would be true that: $P(AG|WH) = \frac{P(WH \cap AG)}{P(WH)} = P(AG)$.

So in general for all independent events,

$$\text{If } P(A|B) = \frac{P(A \cap B)}{P(B)}, \text{then } P(A) = \frac{P(A \cap B)}{P(B)}$$

If you solved for $P(A \cap B)$, then

Probability of <u>Independent</u>
Events A and B
$P(A \cap B) = P(A) \cdot P(B)$

Part 5 - Confirming Independence

By developing a full picture of conditional probability in the previous task, you were able to conclude that events that occur without regard to conditions, independent events, are defined by the equation $P(A \cap B) = P(A) \cdot P(B)$. This equation is known as necessary and sufficient. It works exactly like a biconditional statement: two events A and B are independent if and only if the equation $P(A \cap B) = P(A) \cdot P(B)$ is true.

1. Based on the definition of independence, determine if each set of events below are independent.

   a. $P(A) = 0.45, P(B) = .30, P(A \cap B) = 0.75$

   b. $P(A) = 0.12, P(B) = .56, P(A \cap B) = 0.0672$

   c. $P(A) = \frac{4}{5}, P(B) = \frac{3}{8}, P(A \cap B) = \frac{7}{40}$

   d. $P(A) = \frac{7}{9}, P(B) = \frac{3}{4}, P(A \cap B) = \frac{7}{12}$

2. Determine the missing values so that the events A and B will be independent.

   a. $P(A) = 0.55, P(B) =$________, $P(A \cap B) = 0.1375$

   b. $P(A) =$________, $P(B) = \frac{3}{10}, P(A \cap B) = \frac{1}{7}$

Part 6 – Independence and Inference

With knowledge of probability and statistics, statisticians are able to make statistical inferences about large sets of data. Based upon what you have learned in this unit, you have the knowledge necessary to make basic inferences.

Much of the data collected every 10 years for the Census is available to the public. This data includes a variety of information about the American population at large such as age, income, family background, education history and place of birth. Below you will find three different samples of the Census that looks at comparing different aspects of American life. Your job will be to use your knowledge of conditional probability and independence to make conclusions about the American populace.

1. Gender vs. Income – Has the gender gap closed in the world today? Are men and women able to earn the same amount of money? The table below organizes income levels (per year) and gender.

	Under \$10,000 (L)	Between \$10,000 and \$40,000 (ML)	Between \$40,000 and \$100,000 (MH)	Over \$100,000 (H)	
Male (M)	15	64	37	61	
Female (F)	31	73	14	58	

By finding different probabilities from the table above, make a determination about whether or not income level is affected by gender. Investigate whether your conclusion is true for all income levels. Show all the calculations you use and write a conclusion using those calculations.

$P(M) =$________ $P(F) =$________ $P(L) =$________

$P(H) =$________ $P(MH) =$________ $P(H) =$________

$P(F \cap MH)=$________ $P(M \cap H) =$________ $P(M \cap L) =$________

Are the events being male and making over \$100,000 independent?

Are the events being a male and making less than \$10,000 independent?

2. Suppose you roll two fair, six-sided dice – one is red and one is green. Are the events “sum of 8” and “green die shows a 4” independent?

3. Suppose you roll two fair, six-sided dice – one is red and one is green. Are the events “sum of 7” and “green die shows a 4” independent?

4. When rolling two number cubes:

   What is the probability of rolling a sum that is greater than 7?

   What is the probability of rolling a sum that is odd?

   Are the events, rolling a sum greater than 7, and rolling a sum that is odd, independent?

5. What is the probability of drawing a heart from a standard deck of cards on a second draw, given that a heart was drawn on the first draw and not replaced? Are these events independent or dependent?

Tree Diagrams

We know that modern medicine is rich with biology, chemistry and countless other branches of science. While mathematics does not make a lot of headlines for changing the way we think about our health and well-being, there are many ways in which mathematics is informing and improving the way scientists and doctors approach the world of medicine.

Consider the following data for a group of 1000 women. Of these women, 8 are known to have breast cancer. All 1000 undergo the same test to determine whether or not they have breast cancer. The tests came back positive for 87.5% of those that actually have breast cancer and 7.1% that do not have breast cancer also tested positive.

a) Organize the data in a tree diagram below.

b) Find the following probabilities:

   a. The probability that a woman tests positive.

   b. The probability that a women has breast cancer, given that she tested positive.

c) Let's also look through the lens of a woman who tests negative.

   a. Find the probability that a woman does not have breast cancer given that her test result is negative.

   b. What does this result indicate?

In June 2008, 88% of automobile drivers filled their tanks with regular gasoline, 2% purchased midgrade gas, and 10% bought premium gas. Of those who bought regular gas, 28% paid with a credit card. Of customers who bought midgrade and premium gas, 34% and 42%, respectively, paid with a credit card. Suppose we select a customer at random.

(a) Construct a tree diagram to represent this situation.

(b) Find the probability that the customer paid with a credit card.

(c) Given that the customer paid with a credit card, find the probability that he or she bought premium gas.

A school detects a reason for first time freshmen having a large failing percentage. When they look further into the data, 17% of those that transferred schools in middle school failed. Of those that did not transfer schools in middle school, 8% failed. On average, approximately 24% of our first time freshmen transferred schools in middle school.

(a) What is the probability that a student selected at random failed their freshmen year? Show your work.

(b) What is the probability that a student selected at random who failed their freshmen year, transferred schools in middle school?

Expected Value

1. A commuter must pass through five traffic lights on her way to work and will have to stop at each one that is red. She estimates the probability model for the number of red lights she hits, as shown below:

X=# of red lights	0	1	2	3	4	5
P(X)	0.05	0.25	0.35	0.15	0.15	0.05

How many red lights should she expect to hit each day? What does that number mean?

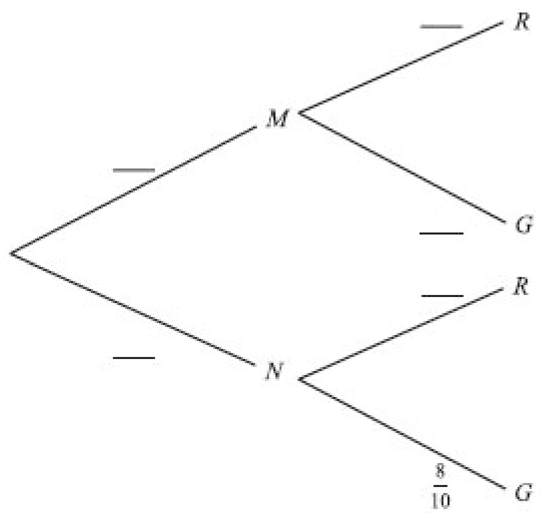
2. A consumer organization inspecting new cars found that many had appearance defects (dents, scratches, paint chips, etc.). While none had more than three of these defects, 7% had three, 11% two, and 21% one defect. Find the expected number of defects in a new car. Explain what this number means.

3. I flip a coin twice and count the number of heads. Find the expected number of heads in the two flips.

IB Probability Questions

1. A game is played, where a die is tossed and a marble selected from a bag.
Bag M contains 3 red marbles (R) and 2 green marbles (G).
Bag N contains 2 red marbles and 8 green marbles.
A fair six-sided die is tossed. If a 3 or 5 appears on the die, bag M is selected (M).
If any other number appears, bag N is selected (N).
A single marble is then drawn at random from the selected bag.

(a) **Copy** and **complete** the probability tree diagram on **your answer sheet.**



(b) (i) Write down the probability that bag M is selected and a green marble drawn from it.

(ii) Find the probability that a green marble is drawn from either bag.

(iii) Given that the marble is green, calculate the probability that it came from Bag M.

(c) A player wins \$2 for a red marble and \$5 for a green marble. What are his expected winnings?

2. Two unbiased 6-sided dice are rolled, a red one and a black one. Let E and F be the events

E : the same number appears on both dice;

F : the sum of the numbers is 10.

Find

(a) P(E);

(b) P(F);

(c) P($E \cup F$).

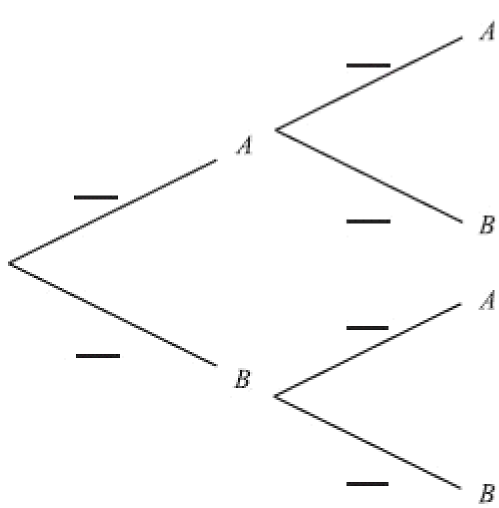
3. Events E and F are independent, with P(E) = $\frac{2}{3}$ and P($E \cap F$) = $\frac{1}{3}$. Calculate

(a) P(F);

(b) P($E \cup F$).

4. A bag contains four apples (A) and six bananas (B). A fruit is taken from the bag and eaten. Then a second fruit is taken and eaten.

(a) Complete the tree diagram below by writing probabilities in the spaces provided.



(b) Find the probability that one of each type of fruit was eaten.

5. Bag A contains 2 red balls and 3 green balls. Two balls are chosen at random from the bag without replacement. Let X denote the number of red balls chosen. The following table shows the probability distribution for X

X	0	1	2
$P(X = x)$	$\frac{3}{10}$	$\frac{6}{10}$	$\frac{1}{10}$

(a) Calculate $E(X)$, the mean number of red balls chosen.

Bag B contains 4 red balls and 2 green balls. Two balls are chosen at random from bag B.

(b) (i) Draw a tree diagram to represent the above information, including the probability of each event.

(ii) Hence find the probability distribution for Y, where Y is the number of red balls chosen.

A standard die with six faces is rolled. If a 1 or 6 is obtained, two balls are chosen from bag A, otherwise two balls are chosen from bag B.

(c) Calculate the probability that two red balls are chosen.

(d) Given that two red balls are obtained, find the conditional probability that a 1 or 6 was rolled on the die.

6. Let A and B be independent events such that $\mathrm{P}(A) = 0.3$ and $\mathrm{P}(B) = 0.8$.

(a) Find $\mathrm{P}(A \cap B)$.

(b) Find $\mathrm{P}(A \cup B)$.

(c) Are A and B mutually exclusive? Justify your answer.

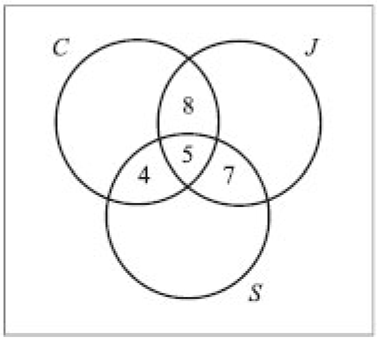
7. In any given season, a soccer team plays 65% of their games at home.
When the team plays at home, they win 83% of their games.
When they play away from home, they win 26% of their games.

The team plays one game.

(a) Find the probability that the team wins the game.

(b) If the team does not win the game, find the probability that the game was played at home.

8. The Venn diagram below shows information about 120 students in a school. Of these, 40 study Chinese (C), 35 study Japanese (J), and 30 study Spanish (S).



A student is chosen at random from the group. Find the probability that the student

(a) studies exactly two of these languages;

(b) studies only Japanese;

(c) does not study any of these languages.

9. There are 20 students in a classroom. Each student plays only one sport. The table below gives their sport and gender.

	Football	**Tennis**	**Hockey**
Female	5	3	3
Male	4	2	3

(a) One student is selected at random.

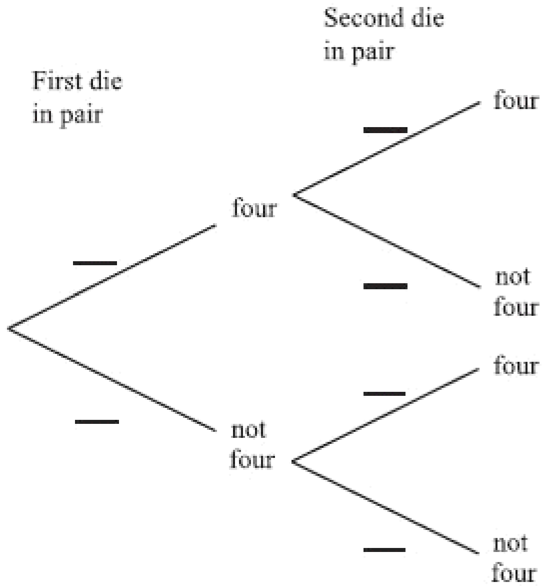
(i) Calculate the probability that the student is a male or is a tennis player.

(ii) Given that the student selected is female, calculate the probability that the student does not play football.

(b) Two students are selected at random. Calculate the probability that neither student plays football.

10. A pair of fair dice is thrown.

(a) Copy and complete the tree diagram below, which shows the possible outcomes.



Let E be the event that **exactly** one four occurs when the pair of dice is thrown.

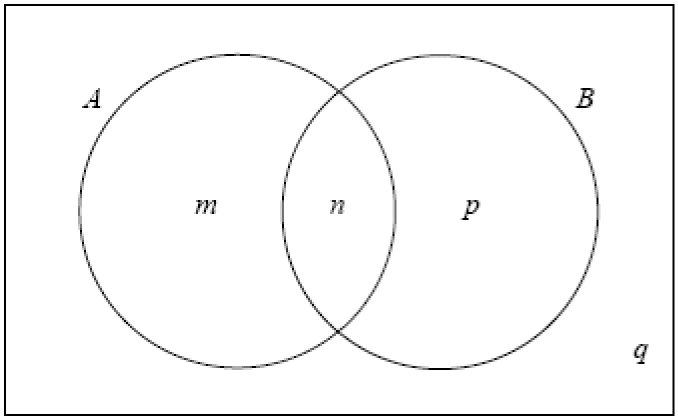
(b) Calculate P(E).

The pair of dice is now thrown five times.

(c) Calculate the probability that event E occurs **exactly** three times in the five throws.

(d) Calculate the probability that event E occurs **at least** three times in the five throws.

11. The Venn diagram below shows events A and B where $\mathrm{P}(A) = 0.3$, $\mathrm{P}(A \cup B) = 0.6$ and $\mathrm{P}(A \cap B) = 0.1$. The values m, n, p and q are probabilities.

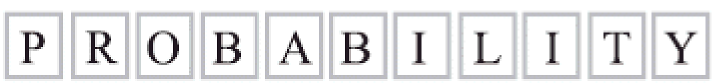


(a) (i) Write down the value of n.

(ii) Find the value of m, of p, and of q.

(b) Find $\mathrm{P}(B')$.

12. The letters of the word PROBABILITY are written on 11 cards as shown below.



Two cards are drawn at random without replacement.
Let A be the event the first card drawn is the letter A.
Let B be the event the second card drawn is the letter B.

(a) Find $\mathrm{P}(A)$.

(b) Find $\mathrm{P}(B \mid A)$.

(c) Find $\mathrm{P}(A \cap B)$.

13. A **four-sided** die has three blue faces and one red face. The die is rolled.

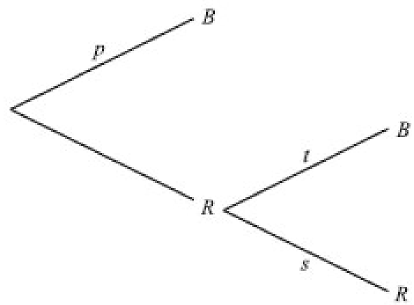
Let B be the event a blue face lands down, and R be the event a red face lands down.

(a) Write down

(i) P (B);

(ii) P (R).

(b) If the blue face lands down, the die is not rolled again. If the red face lands down, the die is rolled once again. This is represented by the following tree diagram, where p, s, t are probabilities.



Find the value of p, of s and of t.

Guiseppi plays a game where he rolls the die. If a blue face lands down, he scores 2 and is finished. If the red face lands down, he scores 1 and rolls one more time. Let X be the total score obtained.

(c) (i) Show that P $(X = 3) = \frac{3}{16}$.

(ii) Find P $(X = 2)$.

(d) (i) Construct a probability distribution table for X.

(ii) Calculate the expected value of X.

(e) If the total score is 3, Guiseppi wins \$10. If the total score is 2, Guiseppi gets nothing.

Guiseppi plays the game twice. Find the probability that he wins exactly \$10.

14. Two standard six-sided dice are tossed. A diagram representing the sample space is shown below.

		Score on second die					
		1	2	3	4	5	6
	1	•	•	•	•	•	•
	2	•	•	•	•	•	•
Score on first die	3	•	•	•	•	•	•
	4	•	•	•	•	•	•
	5	•	•	•	•	•	•
	6	•	•	•	•	•	•

Let X be the sum of the scores on the two dice.

(a) Find

(i) $P(X = 6)$;

(ii) $P(X > 6)$;

(iii) $P(X = 7 \mid X > 5)$.

(b) Elena plays a game where she tosses two dice.

If the sum is 6, she wins 3 points.
If the sum is greater than 6, she wins 1 point.
If the sum is less than 6, she **loses** k points.

Find the value of k for which Elena's expected number of points is zero.

15. In a class of 100 boys, 55 boys play football and 75 boys play rugby. Each boy must play at least one sport from football and rugby.

(a) (i) Find the number of boys who play both sports.

(ii) Write down the number of boys who play only rugby.

(b) One boy is selected at random.

(i) Find the probability that he plays only one sport.

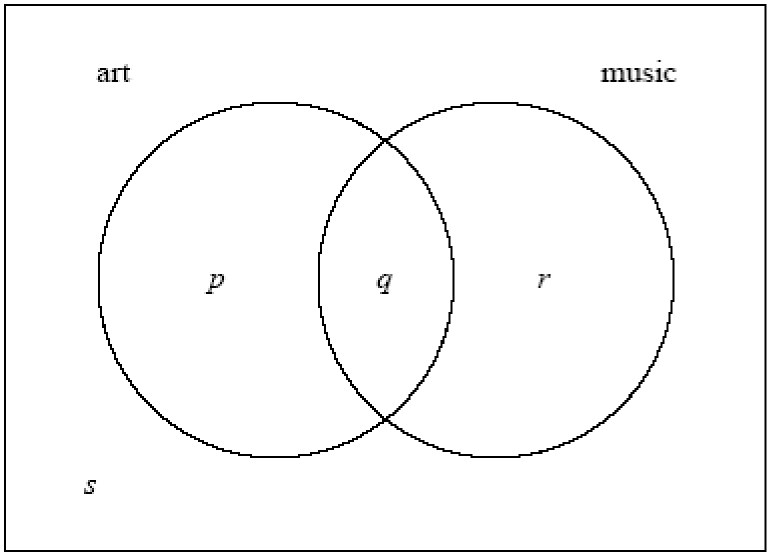
(ii) Given that the boy selected plays only one sport, find the probability that he plays rugby.

Let A be the event that a boy plays football and B be the event that a boy plays rugby.

(c) Explain why A and B are **not** mutually exclusive.

(d) Show that A and B are **not** independent.

16. In a group of 16 students, 12 take art and 8 take music. One student takes neither art nor music. The Venn diagram below shows the events art and music. The values p, q, r and s represent numbers of students.



(a) (i) Write down the value of s.

(ii) Find the value of q.

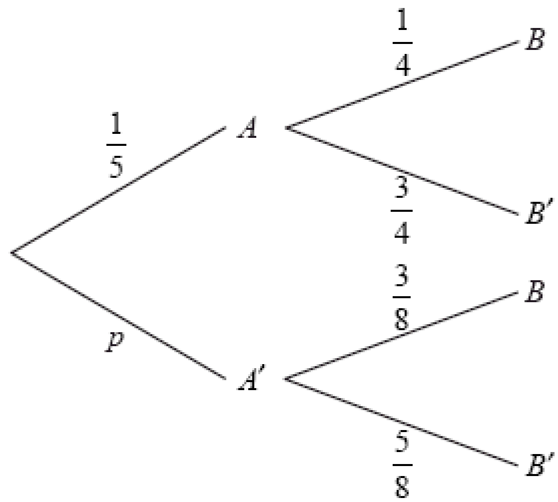
(iii) Write down the value of p and of r.

(b) (i) A student is selected at random. Given that the student takes music, write down the probability the student takes art.

(ii) **Hence**, show that taking music and taking art are **not** independent events.

(c) Two students are selected at random, one after the other. Find the probability that the first student takes **only** music and the second student takes **only** art.

17. The diagram below shows the probabilities for events A and B, with $P(A') = p$.



(a) Write down the value of p.

(b) Find $P(B)$.

(c) Find $P(A' \mid B)$.

18. A company uses two machines, A and B, to make boxes. Machine A makes 60% of the boxes.

80% of the boxes made by machine A pass inspection.
90% of the boxes made by machine B pass inspection.

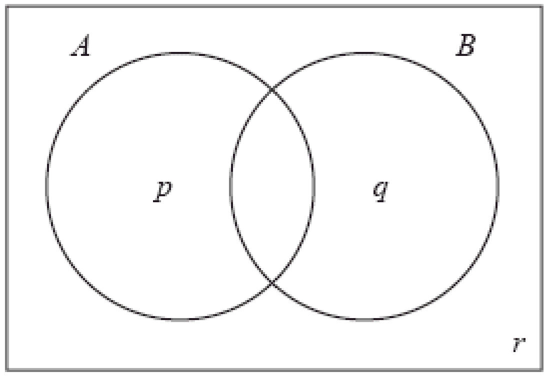
A box is selected at random.

(a) Find the probability that it passes inspection.

(b) The company would like the probability that a box passes inspection to be 0.87.
Find the percentage of boxes that should be made by machine B to achieve this.

19. Consider the events A and B, where $P(A) = 0.5$, $P(B) = 0.7$ and $P(A \cap B) = 0.3$.

The Venn diagram below shows the events A and B, and the probabilities p, q and r.



(a) Write down the value of

(i) p;

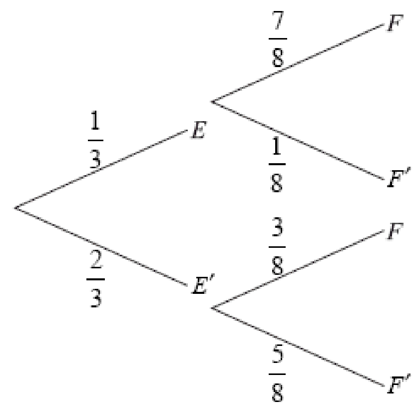
(ii) q;

(iii) r.

(b) Find the value of $P(A \mid B')$.

(c) Hence, or otherwise, show that the events A and B are **not** independent.

20. José travels to school on a bus. On any day, the probability that José will miss the bus is $\frac{1}{3}$.
If he misses his bus, the probability that he will be late for school is $\frac{7}{8}$.
If he does not miss his bus, the probability that he will be late is $\frac{3}{8}$.
Let E be the event "he misses his bus" and F the event "he is late for school".
The information above is shown on the following tree diagram.



(a) Find

(i) $P(E \cap F)$;

(ii) $P(F)$.

(b) Find the probability that

(i) José misses his bus and is not late for school;

(ii) José missed his bus, given that he is late for school.

The cost for each day that José catches the bus is 3 euros. José goes to school on Monday and Tuesday.

(c) **Copy** and complete the probability distribution table.

***X* (cost in euros)**	0	3	6
P (*X*)	$\frac{1}{9}$		

(d) Find the expected cost for José for both days.