

1. Let $f(x) = \log_3 \frac{x}{2} + \log_3 16 - \log_3 4$, for $x > 0$.

(a) Show that $f(x) = \log_3 2x$.

$$\log_3 \frac{x}{2} + \log_3 16 - \log_3 4$$

$$\log_3 8x - \log_3 4$$

$$f(x) = \log_3 2x$$

(b) Find the value of $f(0.5)$ and of $f(4.5)$.

$$f(1/2) = \log_3 (2(1/2)) = \log_3 1 = 0$$

$$f(9/2) = \log_3 (2(9/2)) = \log_3 9 = 2$$

2. (a) Find $\log_2 32$.

$$\log_2 32 = 5$$

(b) Given that $\log_2 \left(\frac{32^x}{8^y} \right)$ can be written as $px + qy$, find the value of p and of q .

$$\log_2 32^x - \log_2 8^y$$

$$x \log_2 32 - y \log_2 8$$

$$5x - 3y$$

$$p = 5$$

$$q = -3$$

3. If $\log_a 2 = x$ and $\log_a 5 = y$, find in terms of x and y , expressions for

(a) $\log_2 5 = \frac{\log_a 5}{\log_a 2} = \frac{y}{x}$

(b) $\log_a 20 = \log_a 5 + \log_a 2^2 = \log_a 5 + 2\log_a 2 = y + 2x$

4. Given that $p = \log_a 5$, $q = \log_a 2$, express the following in terms of p and/or q .

(a) $\log_a 10$ $p + q$

(b) $\log_a 8$ $3q$

(c) $\log_a 2.5$ $p - q$

5. (a) Let $\log_c 3 = p$ and $\log_c 5 = q$. Find an expression in terms of p and q for

(i) $\log_c 15$; $p + q$

(ii) $\log_c 25$. $2q$

(b) Find the value of d if $\log_d 6 = \frac{1}{2}$. $d = 36$

6. Let $\ln a = p$, $\ln b = q$. Write the following expressions in terms of p and q .

(a) $\ln a^3 b$ $3p + q$

(b) $\ln \left(\frac{\sqrt{a}}{b} \right)$ $\frac{1}{2}p - q$

7. Solve the following equations.

(a) $\log_x 49 = 2$ $x = 7$

(b) $\log_2 8 = x$ $x = 3$

(c) $\log_{25} x = -\frac{1}{2}$ $x = \frac{1}{5}$

(d) $\log_2 x + \log_2(x - 7) = 3$

$$\log_2(x^2 - 7x) = 3$$

$$8 = x^2 - 7x$$

$$0 = x^2 - 7x - 8$$

$$0 = (x - 8)(x + 1)$$

$$x = 8, -1$$

$$x = 8$$