Accelerated Geom/Alg 2
Polynomial Test #1 Review

Name ______ Date ______ Block ______

Use synthetic division to find the quotient.

1. $\dfrac{x^4-5x^3+7x^2-8x+1}{x-2}$

$$\begin{array}{r|rrrrr} 2 & 1 & -5 & 7 & -8 & 1 \\ & \downarrow & 2 & -6 & 2 & -12 \\ \hline & 1 & -3 & 1 & -6 & \boxed{-11} \end{array}$$

$\boxed{x^3-3x^2+x-6+\dfrac{-11}{x-2}}$

2. $\dfrac{3x^3-2x+1}{x+3}$

$$\begin{array}{r|rrrr} -3 & 3 & 0 & -2 & 1 \\ & \downarrow & -9 & 27 & -75 \\ \hline & 3 & -9 & 25 & \boxed{-74} \end{array}$$

$\boxed{3x^2-9x+25+\dfrac{-74}{x+3}}$

Use long division to find the quotient.

3. $5x^3-7x^2+2x-4 \div x^2-1$

$$\begin{array}{r} 5x-7 \\ x^2+0x-1 \enclose{longdiv}{5x^3-7x^2+2x-4} \\ -\cancel{5x^3}+0x^2+5x\ \downarrow \\ \hline -\cancel{7x^2}+7x-4 \\ +\cancel{7x^2}-0x\mp 7 \\ \hline 7x-11 \end{array}$$

$\boxed{5x-7+\dfrac{7x-11}{x^2-1}}$

4. $3x^4+8x^3-2x^2-x+3 \div 3x+2$

$$\begin{array}{r} x^3+2x^2-2x+1 \\ 3x+2 \enclose{longdiv}{3x^4+8x^3-2x^2-x+3} \\ -\cancel{3x^4}\mp 2x^3\ \downarrow \\ \hline \cancel{6x^3}-2x^2 \\ -\cancel{6x^3}\mp 4x^2\ \downarrow \\ \hline -\cancel{6x^2}-x \\ +\cancel{6x^2}+4x\ \downarrow \\ \hline 3\cancel{x}+3 \\ -\cancel{3x}\mp 2 \\ \hline 1 \end{array}$$

$\boxed{x^3+2x^2-2x+1+\dfrac{1}{3x+2}}$

5. Use Pascal's Triangle to expand: $(2x-3y)^4$

$1(2x)^4$

$-4(2x)^3(3y)$

$6(2x)^2(3y)^2$

$\boxed{\ldots x^4 - 96x^3y+216x^2y^2 \ldots}$

$+\cancel{7x^2} - 0x \mp 7$

$7x - 11$

$\boxed{5x - 7 + \frac{7x-11}{x^2-1}}$

$-6x^3 \mp 4x$

$-6x$

$+6x^2$

5. Use Pascal's Triangle to expand: $(2x-3y)^4$

$1(2x)^4$

$-4(2x)^3(3y)$

$6(2x)^2(3y)^2$

$-4(2x)(3y)^3$

$1\ (3y)^4$

$= \boxed{16x^4 - 96x^3y + 216x^2y^2 - 216xy^3 + 81y^4}$

x^3

6. Use Pascal's Triangle to expand: $(8-x)^3$

$1(8)^3$

$-3(8)^2(x)$

$3(8)(x)^2$

$-1\ (x)^3$

$= \boxed{512 - 192x + 24x^2 - x^3}$

7. Find the coefficient of a^3b^4 in the expansion of $(4a+b)^7$.

${}_7C_4$ $\quad \underline{35}(4a)^3b^4 = 2240a^3b^4$ $\quad 2240$

8. Consider the expansion of $(x+3)^{10}$.

(a) Write down the number of terms in this expansion. 11

(b) Find the term containing x^2.

$\underline{45}(x)^2(3)^8 = \boxed{295245x^2}$

${}_{10}C_8$

9. Find the coefficient of x^4 in the expansion of $(2-x)^6$.

${}_6C_4$ $\quad \underline{15}(2)^2(x)^4$ $\quad 60x^4$ $\quad 60$

10. Find the coefficient of x^5 in the expansion of $(3x-2)^8$.

${}_8C_3$ $\quad -\underline{56}(3x)^5(2)^3 = -108864x^5$ $\quad -108864$

$(2\ \ x)^7$

$_8C_3$

11. Determine the constant term in the expansion of $\cancel{\left(\frac{2}{x}+x^2\right)^7}$. $\left(\frac{2}{x^2}+x\right)^9$

$\left(\frac{2}{x^2}\right)^9$

$\left(\frac{2}{x^2}\right)^8(x)$

$\left(\frac{2}{x^2}\right)^7(x)^2$

$\left(\frac{2}{x^2}\right)^6(x)^3$

$\left(\frac{2}{x^2}\right)^5(x)^4$

$\left(\frac{2}{x^2}\right)^4(x)^5$

$_9C_6$ $\underline{84}\left(\frac{2}{x^2}\right)^3(x)^6 = 84\left(\cancel{\frac{8}{x^6}}\right)\cancel{x^6} = 672$

12. Find the coefficient of a^8b^4 in the expansion of $(a+b)^{12}$.

$_{12}C_4$ $\underline{495}(a)^8(b)^4$ 495

13. Find the term containing x^{10} in the expansion of $(5+2x^2)^7$.

$_7C_5$ $\underline{21}(5)^2(2x^2)^5 = 16800x^{10}$

14. Given that $(2-\sqrt{5})^3 = p + q\sqrt{5}$ where p and q are integers, find

$1(2)^3$

$-3(2)^2(\sqrt{5})$

$3(2)(\sqrt{5})^2 = 8 - 12\sqrt{5} + 30 - 5\sqrt{5}$

$-1 \quad (\sqrt{5})^3$ $= 38 - 17\sqrt{5}$

(a) $p = 38$

(b) $q = -17$

15. (a) Expand $\left(e+\frac{2}{e}\right)^4$ in terms of e.

$$1\,(e)^4$$
$$4\,(e)^3\left(\tfrac{2}{e}\right)$$
$$6\,(e)^2\left(\tfrac{2}{e}\right)^2 \quad = e^4+8e^2+24+\frac{32}{e^2}+\frac{16}{e^4}$$
$$4\,(e)\left(\tfrac{2}{e}\right)^3$$
$$1\quad\left(\tfrac{2}{e}\right)^4$$

(b) Express $\left(e+\frac{2}{e}\right)^4+\left(e-\frac{2}{e}\right)^4$ as the sum of three terms.

$$e^4+\cancel{8e^2}+24+\cancel{\frac{32}{e^2}}+\frac{16}{e^4}+e^4-\cancel{8e^2}+24-\cancel{\frac{32}{e^2}}+\frac{16}{e^4}$$

$$\boxed{2e^4+48+\frac{32}{e^4}}$$

16. (a) Expand $(x-3)^4$ and simplify your result.

$$1\,(x)^4$$
$$-4\,(x)^3(3)$$
$$6\,(x)^2(3)^2 \quad = x^4-12x^3+54x^2-108x+81$$
$$-4\,(x)(3)^3$$
$$1\quad(3)^4$$

(b) Find the term in x^3 in $(2x+5)(x-3)^4$.

	x^4	$-12x^3$	$54x^2$	$-108x$	81
$2x$	$8x^5$	$-24x^4$	$108x^3$	$-216x^2$	$162x$
5	$5x^4$	$-60x^3$	$270x^2$	$-540x$	405

$48x^3$

Find the inverse of the following functions.

17. $f(x) = 8x - 9$

$x = 8y - 9$

$x + 9 = 8y$

$\frac{x+9}{8} = y$

$\boxed{f^{-1}(x) = \frac{x+9}{8}}$

18. $g(x) = 5(x - 15)$

$x = 5(y - 15)$

$\frac{x}{5} = y - 15$

$\frac{x}{5} + 15 = y$

$\boxed{g^{-1}(x) = \frac{x}{5} + 15}$

19 $h(x) = 6x + 24$

$x = 6y + 24$

$x - 24 = 6y$

$\frac{x-24}{6} = y$

$\boxed{h^{-1}(x) = \frac{x-24}{6} \text{ OR } \frac{x}{6} - 4}$

20. $j(x) = (x - 9)^2 + 1$

$x = (y-9)^2 + 1$

$x - 1 = (y-9)^2$

$\sqrt{x-1} = y - 9$

$\sqrt{x-1} + 9 = y$

$\boxed{j^{-1}(x) = \sqrt{x-1} + 9}$

21. $k(x) = \sqrt{2x - 5} - 3$

$x = \sqrt{2x-5} + 3$

$x + 3 = \sqrt{2x-5}$

$(x+3)^2 = 2x - 5$

$(x+3)^2 + 5 = 2x$

$\frac{(x+3)^2 + 5}{2} = x$

$\boxed{k^{-1}(x) = \frac{(x+3)^2 + 5}{2}}$

22 $m(x) = 4x^2 - 1$

$x = 4y^2 - 1$

$x + 1 = 4y^2$

$\frac{x+1}{4} = y^2$

$\sqrt{\frac{x+1}{4}} = y$

$\boxed{m^{-1}(x) = \frac{\sqrt{x+1}}{2}}$

Perform the following operations for the functions:

$f(x) = 2x - 3$ $\qquad g(x) = 2x^2 - 7x + 4$

$j^{-1}(x) = \sqrt{x-1}+9$

$k^{-1}(x) = \frac{(x+3)^2+5}{2}$

$m^{-1}(x) = \frac{\sqrt{x+1}}{2}$

Perform the following operations for the functions:

$f(x) = 2x - 3 \qquad g(x) = 2x^2 - 7x + 4$

23. $f(g(x))$

$2(2x^2-7x+4)-3$

$4x^2-14x+8-3$

$\boxed{f(g(x))=4x^2-14x+5}$

	$2x$	-3
$2x$	$4x^2$	$-6x$
-3	$-6x$	9

24. $g(f(x))$

$2(2x-3)^2-7(2x-3)+4$

$2(4x^2-12x+9)-14x+21+4$

$8x^2-24x+18-14x+25$

$\boxed{g(f(x))=8x^2-38x+43}$

25. $f(f(x))$

$2(2x-3)-3$

$4x-6-3$

$\boxed{f(f(x))=4x-9}$

26. $g(g(x))$

$2(2x^2-7x+4)^2-7(2x^2-7x+4)+4$

	$2x^2$	$-7x$	4
$2x^2$	$4x^4$	$-14x^3$	$8x^2$
$-7x$	$-14x^3$	$49x^2$	$-28x$
4	$8x^2$	$-28x$	16

$2(4x^4-28x^3+65x^2-56x+16)-14x^2+49x-28+4$

$8x^4-56x^3+130x^2-112x+32-14x^2+49x-28+4$

$\boxed{g(g(x))=8x^4-56x^3+116x^2-63x+8}$