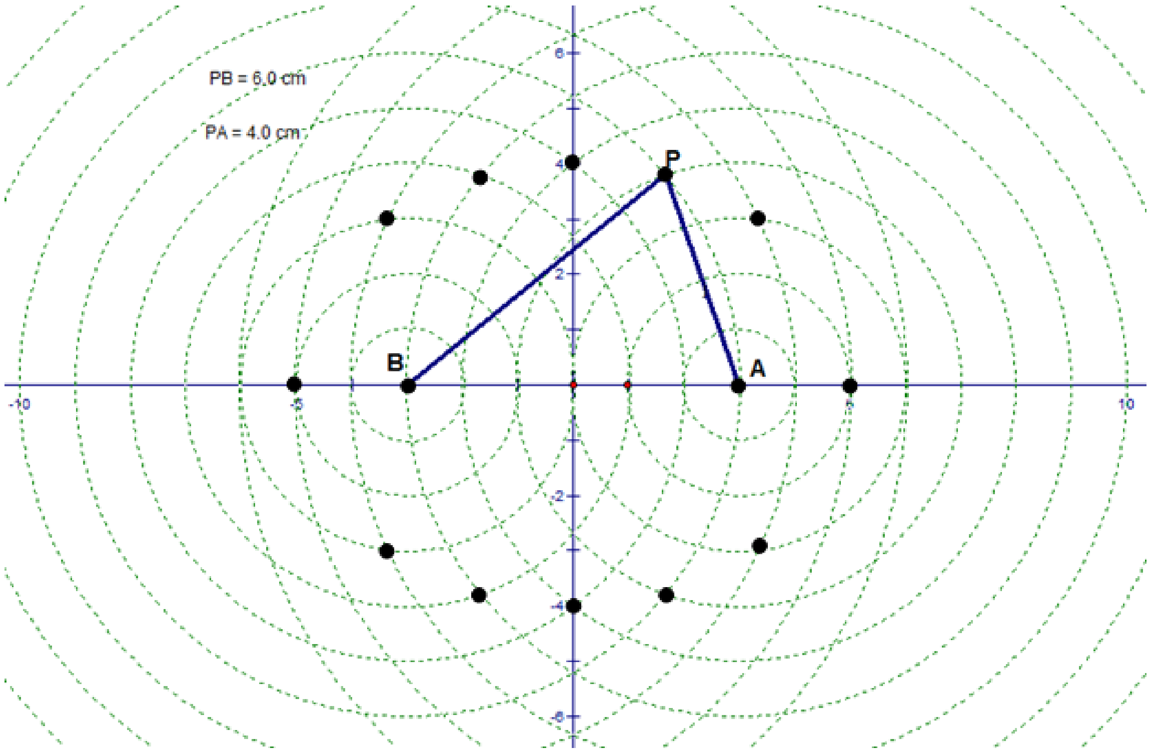
<u>Is It Really an Ellipse?</u>

Standards Addressed
MGSE9-12.G.GPE.3 Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.

An ellipse is the set of all points P in a plane where the sum of the distances from two fixed points (foci) remains constant. In the drawing shown, A and B are the fixed focus points and P is any point on the ellipse where |PA + PB| = 10. Using the two sets of equally spaced circles centered at A and B, check the distances of each of the points marked from foci A and B, are the sums |PA + PB| all equal to 10?

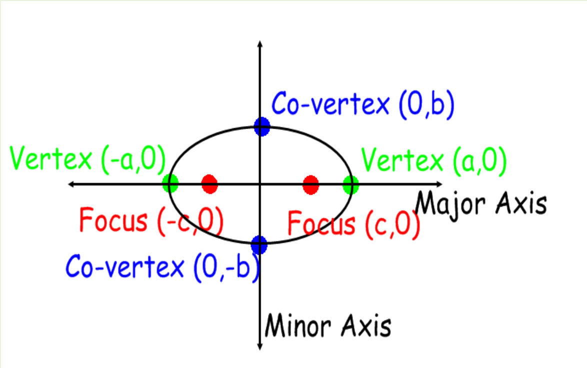


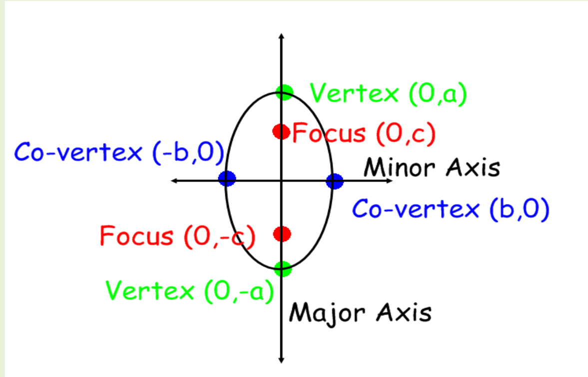
The midpoint of $\overline{AB}$ is the center of the ellipse. For the ellipse shown, the center is (0, 0). The vertices occur at the points (5, 0), (-5, 0), (0, 4), and (0, -4). The segments joining the vertices lying along the x-axis is 10 units long; and, the axis joining the vertices along the y-axis is 8 units long. The longer axis is called the major axis and the shorter axis is called the minor axis.

Using the distance formula and the geometric definition of an ellipse, |PA + PB| = 10, makes it possible to find to write an equation of the ellipse in both general form and standard form.
Focus point A is located at (3, 0) and B is located at (-3, 0). The distance formula gives the following distances. As you work through the following procedures, describe what is being done in each step.

PA = $\sqrt{(x-3)^2+(y-0)^2}$ and PB = $\sqrt{(x+3)^2+(y-0)^2}$	apply the distance formula
PA+PB = 10	given
$\sqrt{(x-3)^2+(y-0)^2}+\sqrt{(x+3)^2+(y-0)^2} = 10$	substitution
$\sqrt{(x-3)^2+(y-0)^2} = 10 - \sqrt{(x+3)^2+(y-0)^2}$	separate the radical expression
$(\sqrt{(x-3)^2+(y-0)^2})^2 = (10-\sqrt{(x+3)^2+(y-0)^2})^2$	square both sides of the equation
$(x-3)^2 + y^2 = 100 - 20\sqrt{(x+3)^2+y^2} + (x+3)^2 + y^2$	begin to expand the right side
$x^2 - 6x + 9 + y^2 = 100 - 20\sqrt{(x+3)^2+y^2} + x^2 + 6x + 9 + y^2$	expand the binomials
$-12x - 100 = -20+\sqrt{(x+3)^2+y^2}$	simplify
$3x + 25 = 5\sqrt{(x+3)^2+y^2}$	divide both sides by -4
$(3x + 25)^2 = \left(5\sqrt{(x+3)^2+y^2}\right)^2$	square again to remove the radical
$9x^2 + 150x + 625 = 25((x+3)^2+y^2)$	expand the binomials
$9x^2 + 150x + 625 = 25(x^2 + 6x + 9 + y^2)$	continue to expand the binomials
$9x^2 + 150x + 625 = 25x^2 + 150x + 225 + 25y^2$	multiply by 25
$16x^2 + 25y^2 = 400$	combine like terms
$\frac{16}{400}x^2 + \frac{25}{400}y^2 = 1$	divide by 400
$\frac{x^2}{25}+\frac{y^2}{16}=1$	simplify to standard form of an ellipse

Summary of Ellipse Information





$$\mathbf{Ax^2 + Cy^2 + Dx + Ey + F = 0}$$

where A ≠ C and AC > 0

$$\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1 \qquad\qquad \frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$$

center: (h, k)

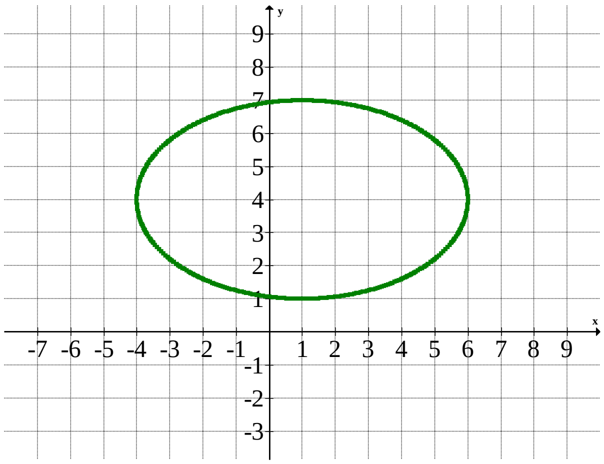
$a^2 > b^2$; $c^2 = a^2 - b^2$

(NOTE: **a** is always the largest number)

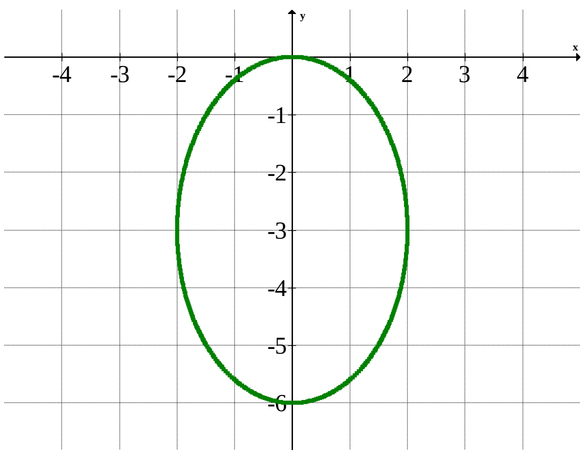
major axis: horizontal	major axis: vertical
length major axis: 2a	length major axis: 2b
length minor axis: 2b	length minor axis: 2a
foci: (h ± c, k)	foci: (h, k ± c)
vertices: (h ± a, k)	vertices: (h, k ± a)
co-vertices: (h, k ± b)	co-vertices: (h ± b, k)

1. Write an equations in both standard and general forms for each ellipse.

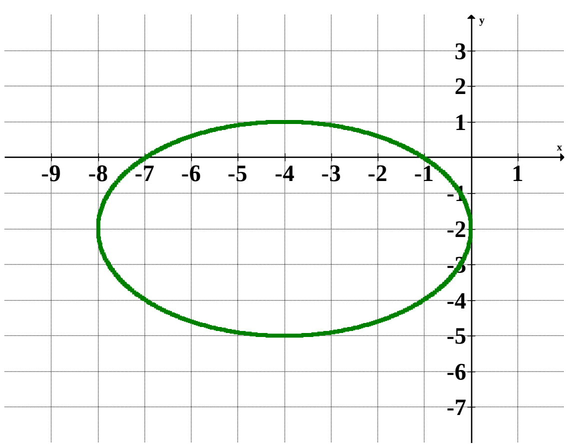
a.



b.

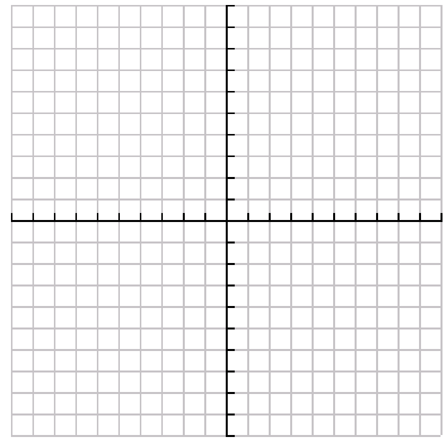


c.

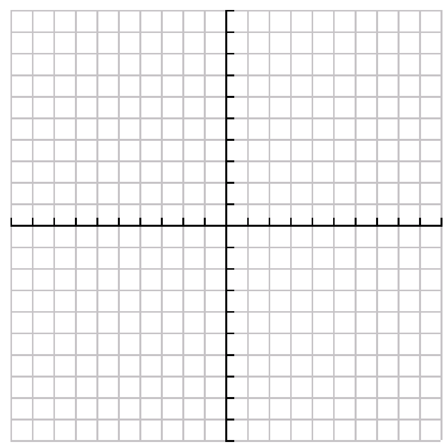


2. Graph each ellipse. Identify the vertices, co-vertices, and focus points. Find the lengths of the major and minor axes.

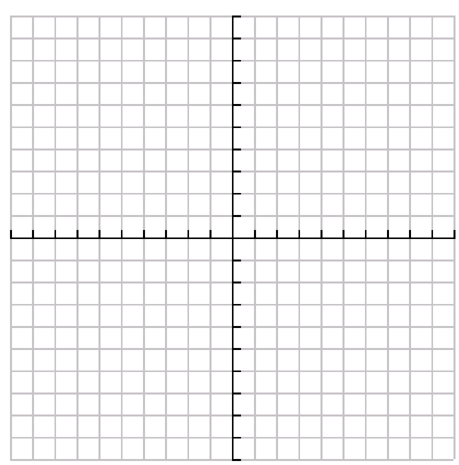
a. $\frac{x^2}{49}+\frac{y^2}{81}=1$



b. $4(x+2)^2 + 9(y-1)^2 = 144$

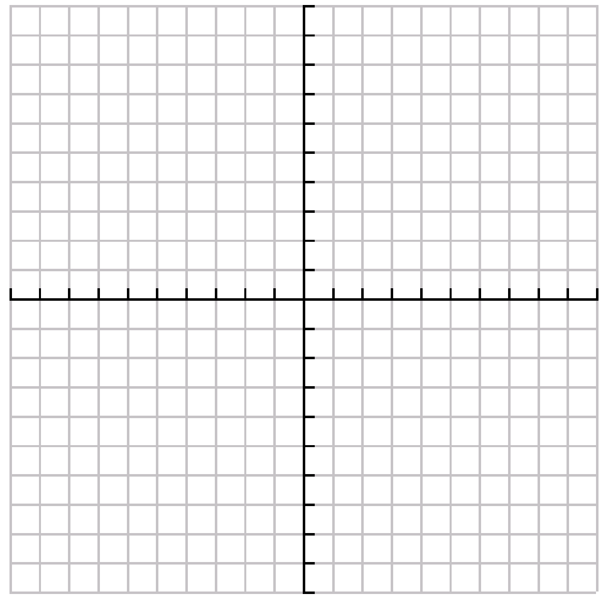


c. $4(x - 3)^2 + 25(y + 2)^2 = 100$

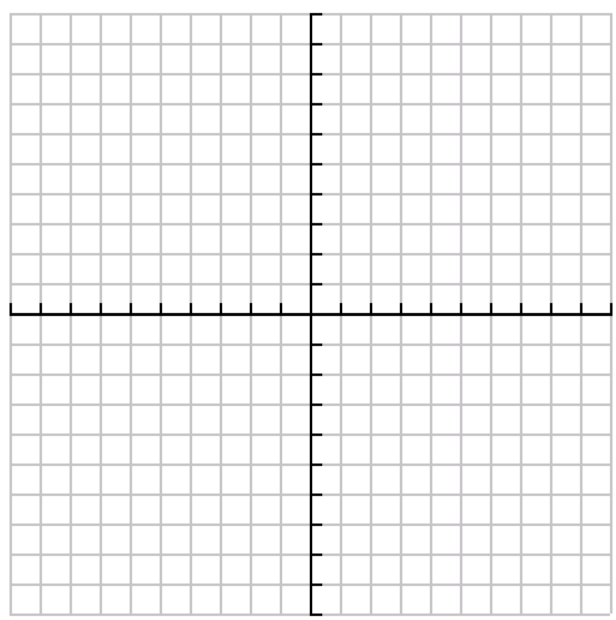


3. Use completing the square to change each general form ellipse to standard form and graph.

a. $9x^2 + 4y^2 + 8y - 32 = 0$



b. $x^2 + 4y^2 + 6x - 8y - 3 = 0$

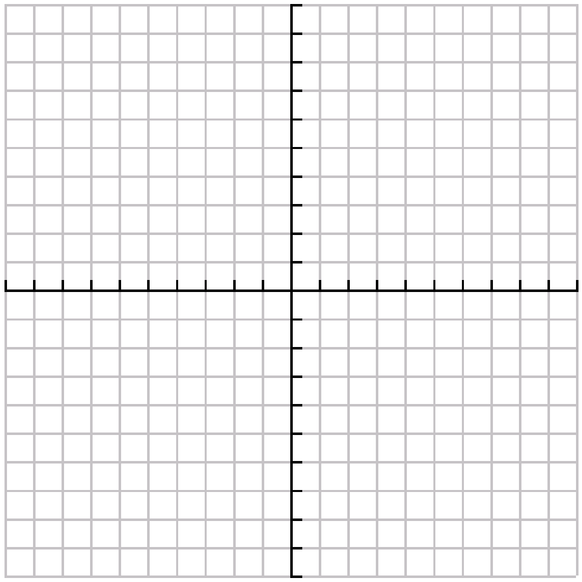


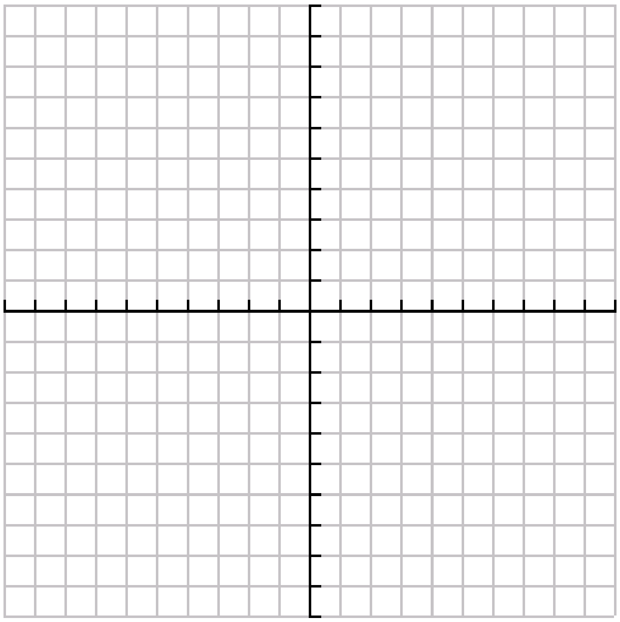
Ellipse Practice

Write the equation of the ellipse in standard form. Identify the vertices, co-vertices, and foci and sketch the graph.

1. $25x^2 + y^2 - 100x - 2y + 76 = 0$

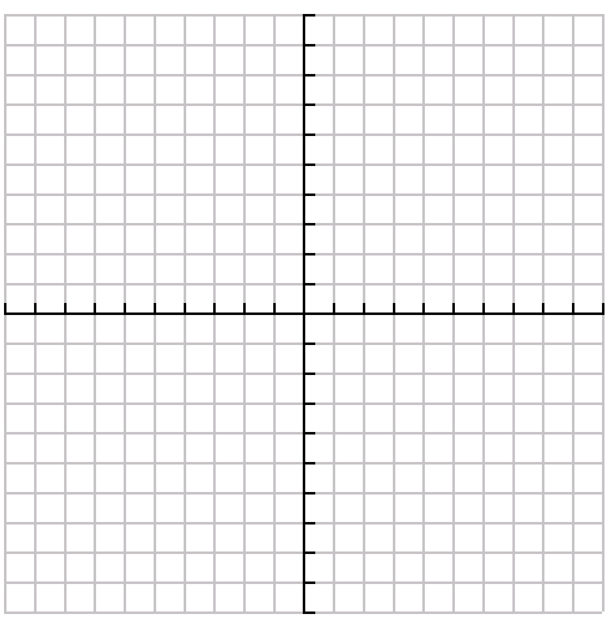
2. $x^2 + 4y^2 - 10x - 40y + 121 = 0$

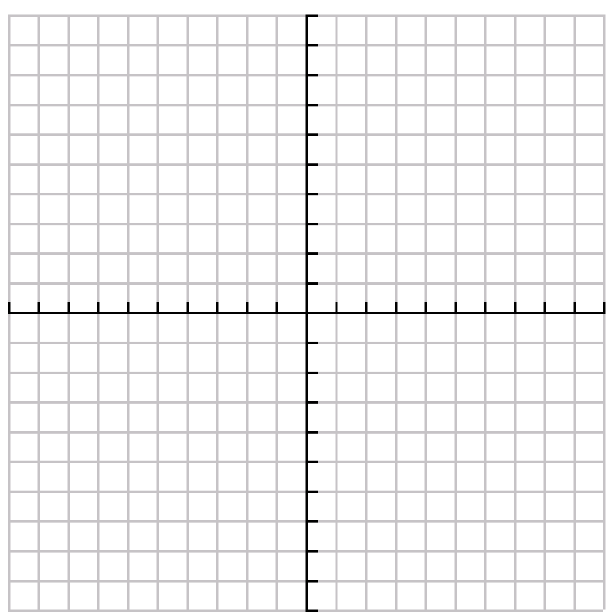




3. $x^2 + 36y^2 - 16x - 72y + 64 = 0$

4. $4x^2 + y^2 - 48x - 4y + 48 = 0$





5. Write the equation of the ellipse with the given characteristics:

   Vertices (5,1) (-3,1) and Foci $(1 - 2\sqrt{3}, 1)$ $(1 + 2\sqrt{3}, 1)$

6. Write the equation of the ellipse with the given characteristics:

   Co-Vertices (-4,2) (2,2) and Foci (-1,6) (-1,-2)