Solving Radical Equations:

MGSE9-12.A.REI.2 Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

A **radical equation** is an equation involving square roots, cubed roots, or nth (any) roots of polynomial expressions.

Remember also that the **inverse** function of a **radical** involves **raising an expression to a power** equal to the index of the radical.

1. Find the inverse of the following functions:

a) $f(x) = (x-4)^3$

$x = (y-4)^3$

$\sqrt[3]{x} = y - 4$

$\boxed{f^{-1}(x) = \sqrt[3]{x} + 4}$

b) $g(x) = \sqrt{x+7}$

$x = \sqrt{y+7}$

$x^2 = y + 7$

$\boxed{g^{-1}(x) = x^2 - 7}$

If $A = B^n$, then $\sqrt[n]{A} = B$. Conversely, if $\sqrt[n]{A} = B$ then $A = B^n$.

There are two more useful properties involving equations that have radicals or exponents on both sides of the equation.

If $(A)^n = (B)^n$, then $A = B$, and If $\sqrt[n]{A} = \sqrt[n]{B}$, then $A = B$.

2. Solve each equation below.

a) $\sqrt[7]{x+3} = \sqrt[7]{15}$

$x + 3 = 15$

$\boxed{x = 12}$

b) $(2x-8)^3 = x^3$

$2x - 8 = x$

$\boxed{x = 8}$

If $(A)^n = (B)^n$, then $A = B$, and If $\sqrt[n]{A} = \sqrt[n]{B}$, then $A = B$.

3. Of course, we may have to work a little to get equations in that form. If we know that $\sqrt[4]{2x-1} - \sqrt[4]{15} = 0$, how can we rewrite the equation to match one of the properties above?

Solve. $\sqrt[4]{2x-1} = \sqrt[4]{15}$

$2x-1=15$

$2x=16$

$\boxed{x=8}$

4. Sometimes solving a radical equation can produce **extraneous solutions**. An extraneous solution is: a solution found when solving an equation, but is NOT a valid solution.

5. Let's look at a simple example first. Take the equation $\sqrt{x} = -3$.

a. Solve $\sqrt{x} = -3$ algebraically.

$(\sqrt{x})^2 = (-3)^2$

$x = 9$

b. How can I check my answer?

$\sqrt{9} = 3$, $\sqrt{9} \neq -3$

c. Let's try solving by graphing. Graph $\sqrt{x}$ into y_1 and -3 into y_2.

Where would I find the solution on the graph?

where the two functions intersect

What do you notice? What is the solution to this equation?

they never intersect, No Solution

6. Let's practice a few more. Do either of these equations have extraneous solutions?

a. $\sqrt[3]{5x+1}+6=2$

$\sqrt[3]{5x+1}=-4$

$5x+1=-64$

$5x=-65$

$\boxed{x=-13}$ ✓

b. $x-3=\sqrt{30-2x}$

$x^2-6x+9=30-2x$

$x^2-4x-21=0$

$(x+3)(x-7)=0$

$x=-3, 7$

extraneous

7. Can you make a general statement as to when a radical equation may have an extraneous solution? Can it happen regardless of the root (index)?

even

Solving Radical Equations- *Solve each of the following radical equations.*

[1] $\sqrt[4]{6x-5}=\sqrt[4]{x+10}$

$6x-5=x+10$

$5x=15$

$\boxed{x=3}$

[2] $\sqrt[3]{6x-5}-\sqrt[3]{x+10}=0$

$6x-5=x+10$

$5x=15$

$\boxed{x=3}$

[3] $2\sqrt[3]{10-3x}=\sqrt[3]{2-x}$

$8(10-3x)=2-x$

$80-24x=2-x$

$78=23x$

$\boxed{x=\frac{78}{23}}$

[4] $\sqrt{3x+5}+2\sqrt{x}=0$

$3x+5=4x$

$x=5$

extraneous

[5] $12\sqrt[4]{x-1}+10=4$

$12\sqrt[4]{x-1}=-6$

$\sqrt[4]{x-1}=-\frac{1}{2}$

$x-1=\frac{1}{16}$

$\boxed{x=\frac{17}{16}}$ extraneous

[7] $\sqrt[3]{3x+1}+5=3$

$\sqrt[3]{3x+1}=-2$

$3x+1=-8$

$3x=-9$

$\boxed{x=-3}$

[9] $-2\sqrt[5]{2x-1}+4=0$

$-2\sqrt[5]{2x-1}=-4$

$\sqrt[5]{2x-1}=2$

$2x-1=32$

$2x=33$ $\boxed{x=16.5}$

[11] $\sqrt{3x+7}=x-1$

$3x+7=x^2-2x+1$

$x^2-5x-6=0$

$(x-6)(x+1)=0$

$\boxed{x=6,-1}$ extraneous

[6] $n=\sqrt{6+5n}$

$n^2=6+5n$

$n^2-5n-6=0$

$(n-6)(n+1)=0$

$\boxed{n=6,-1}$ extraneous

[8] $\sqrt[3]{x+40}=-5$

$x+40=-125$

$\boxed{x=-165}$

[10] $\sqrt{6-r}=r$

$6-r=r^2$

$r^2+r-6=0$

$(r+3)(r-2)=0$

$\boxed{r=2,-3}$ extraneous

[12] $\sqrt{4x-7}+2=5$

$\sqrt{4x-7}=3$

$4x-7=9$

$4x=16$

$\boxed{x=4}$

[13] $\sqrt[4]{2x}-13=-9$

$\sqrt[4]{2x}=4$

$2x=256$

$\boxed{x=128}$

[14] $3\sqrt{x+6}+5=14$

$\sqrt{x+6}=3$

$x+6=9$

$\boxed{x=3}$

[15] $\sqrt{2x-1}+5=2$

$\sqrt{2x-1}=-3$

$2x-1=9$

$2x=10$

$\boxed{x=5 \leftarrow \text{extraneous}}$

[16] $\sqrt{50-5x}=x-10$

$50-5x=x^2-20x+100$

$x^2-15x+50=0$

$(x-5)(x-10)=0$

$\boxed{x=5,10}$

extraneous

Solving Radical Equations (2):

Solve the equation.

1) $\sqrt{x-12}=9$

$x-12=81$

$\boxed{x=93}$

2) $\sqrt[3]{2x+1}-3=0$

$\sqrt[3]{2x+1}=3$

$2x+1=27$

$2x=26$

$\boxed{x=13}$

3) $-5\sqrt{x+7}=25$

$\sqrt{x+7}=-5$

$x+7=25$

$\boxed{x=18}$

extraneous

4) $\sqrt[4]{2x+6}=2$

$2x+6=16$

$2x=10$

$\boxed{x=5}$

5) $3 = \frac{1}{4}\sqrt{3x + 30}$

$12 = \sqrt{3x+30}$

$144 = 3x+30$

$114 = 3x$

$\boxed{x=38}$

6) $-3 = 2\sqrt{x-7} + 7$

$-10 = 2\sqrt{x-7}$

$-5 = \sqrt{x-7}$

$25 = x-7$

$\boxed{x=32 \leftarrow \text{extraneous}}$

7) $\sqrt{4x+12} = \sqrt{6x}$

$4x+12 = 6x$

$12 = 2x$

$\boxed{x=6}$

8) $5\sqrt{x-1} = \sqrt{x+1}$

$25(x-1) = x+1$

$25x - 25 = x+1$

$24x = 26$

$\boxed{x = \frac{13}{12}}$

9) $\sqrt[3]{4x} = \sqrt[3]{x+7}$

$4x = x+7$

$3x = 7$

$\boxed{x = \frac{7}{3}}$

10) $x + 3 = \sqrt{x+5}$

$x^2+6x+9 = x+5$

$x^2+5x+4 = 0$

$(x+4)(x+1) = 0$

$x = -4, -1$

extraneous

11) $\sqrt{3x+13} + 3 = 2x$

$\sqrt{3x+13} = 2x-3$

$3x+13 = 4x^2-12x+9$

$4x^2-15x-4 = 0$

	x	-4
$4x$	$4x^2$	$-16x$
1	x	-4

$(4x+1)(x-4) = 0$

$x = 4, -\frac{1}{4}$

extraneous

12) $\sqrt{x+8} - x = -4$

$\sqrt{x+8} = x-4$

$x+8 = x^2-8x+16$

$x^2-9x+8 = 0$

$(x-8)(x-1) = 0$

$x = 8, 1$

extraneous