Solving Radical Equations:

MGSE9-12.A.REI.2 Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

A **radical equation** is an equation involving square roots, cubed roots, or n^{th} (any) roots of polynomial expressions.

Remember also that the **inverse** function of a **radical** involves **raising an expression to a power** equal to the index of the radical.

1. Find the inverse of the following functions:

a) $f(x) = (x-4)^3$

b) $g(x) = \sqrt{x+7}$

If $A = B^n$, then $\sqrt[n]{A} = B$. Conversely, if $\sqrt[n]{A} = B$ then $A = B^n$.

There are two more useful properties involving equations that have radicals or exponents on both sides of the equation.

If $(A)^n = (B)^n$, then $A = B$, and If $\sqrt[n]{A} = \sqrt[n]{B}$, then $A = B$.

2. Solve each equation below.

a) $\sqrt[7]{x+3} = \sqrt[7]{15}$

b) $(2x-8)^3 = x^3$

$$\text{If } (A)^n = (B)^n, \text{ then } A = B, \text{ and If } \sqrt[n]{A} = \sqrt[n]{B}, \text{ then } A = B.$$

3. Of course, we may have to work a little to get equations in that form. If we know that $\sqrt[4]{2x-1} - \sqrt[4]{15} = 0$, how can we rewrite the equation to match one of the properties above?

Solve.

4. Sometimes solving a radical equation can produce **extraneous solutions**. An extraneous solution is:

5. Let's look at a simple example first. Take the equation $\sqrt{x} = -3$.

a. Solve $\sqrt{x} = -3$ algebraically.

b. How can I check my answer?

c. Let's try solving by graphing. Graph $\sqrt{x}$ into y_1 and -3 into y_2.

Where would I find the solution on the graph?

What do you notice? What is the solution to this equation?

6. Let's practice a few more. Do either of these equations have extraneous solutions?

a. $\sqrt[3]{5x+1}+6=2$

b. $x-3=\sqrt{30-2x}$

7. Can you make a general statement as to when a radical equation may have an extraneous solution? Can it happen regardless of the root (index)?

<u>Solving Radical Equations</u>- *Solve each of the following radical equations.*

[1] $\sqrt[4]{6x-5}=\sqrt[4]{x+10}$

[2] $\sqrt[3]{6x-5}-\sqrt[3]{x+10}=0$

[3] $2\sqrt[3]{10-3x}=\sqrt[3]{2-x}$

[4] $\sqrt{3x+5}+2\sqrt{x}=0$

[5] $12\sqrt[4]{x-1}+10=4$

[6] $n=\sqrt{6+5n}$

[7] $\sqrt[3]{3x+1}+5=3$

[8] $\sqrt[3]{x+40}=-5$

[9] $-2\sqrt[5]{2x-1}+4=0$

[10] $\sqrt{6-r}=r$

[11] $\sqrt{3x+7}=x-1$

[12] $\sqrt{4x-7}+2=5$

[13] $\sqrt[4]{2x}-13=-9$

[14] $3\sqrt{x+6}+5=14$

[15] $\sqrt{2x-1}+5=2$

[16] $\sqrt{50-5x}=x-10$

Solving Radical Equations (2):

Solve the equation.

1) $\sqrt{x-12}=9$

2) $\sqrt[3]{2x+1}-3=0$

3) $-5\sqrt{x+7}=25$

4) $\sqrt[4]{2x+6}=2$

5) $3 = \frac{1}{4}\sqrt{3x + 30}$

6) $-3 = 2\sqrt{x - 7} + 7$

7) $\sqrt{4x + 12} = \sqrt{6x}$

8) $5\sqrt{x - 1} = \sqrt{x + 1}$

9) $\sqrt[3]{4x} = \sqrt[3]{x + 7}$

10) $x + 3 = \sqrt{x + 5}$

11) $\sqrt{3x + 13} + 3 = 2x$

12) $\sqrt{x + 8} - x = -4$