## Circles

**Standards Addressed**

**MGSE9-12.G.GPE.1** Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.

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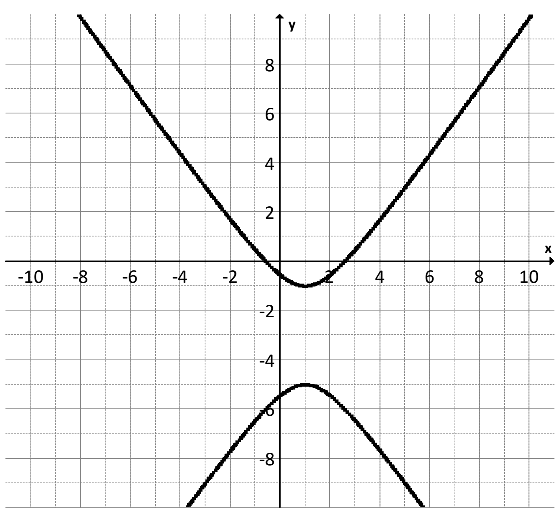
You began this unit by investigating the four different conic sections that are formed when a cone is cut by a plane. Up to this point, much of your learning has focused on the most famous of these, the circle. In order to begin work on the ellipse, hyperbola and parabola, we introduce what is known as the General Form of a Conic Section. All conics can be written with the same structure:

$$Ax^2 + Bxy + Cy^2 + Dx + Ey + F = 0$$

In this task you will be investigating how the different values of *A, B, C, D, E* and *F* determine which conic section is being modeled. In all of the forms of conics that you will be investigating and using, the *B* term is always zero. Thus we will be considering an abbreviated form of the general form:

$$Ax^2 + Cy^2 + Dx + Ey + F = 0$$

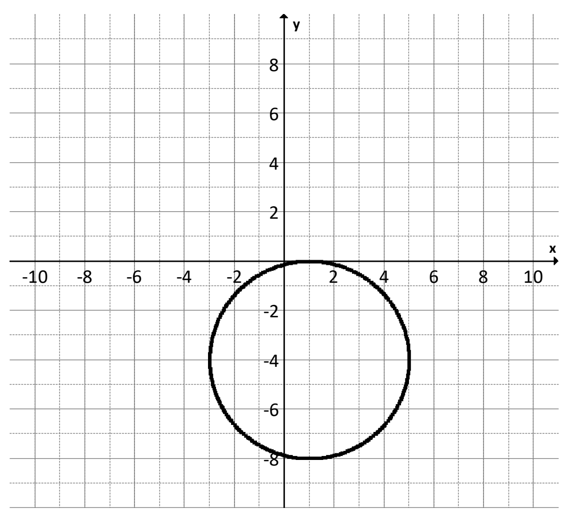
Each of the conic sections below is shown with its equation in general form. Identify each conic section. After doing so, look at each different group of conic section and look for patterns in their general forms. Write any patterns or other conclusions you find in the boxes on the bottom of the last page.




1. Equation:

$$-2x^2 + y^2 + 4x + 6y + 3 = 0$$

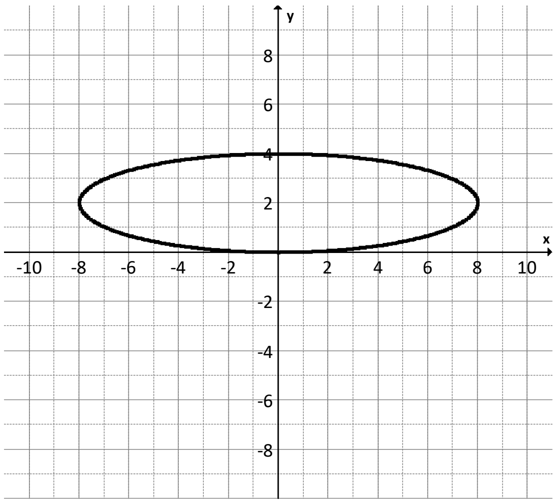
Conic Section:




2. Equation:

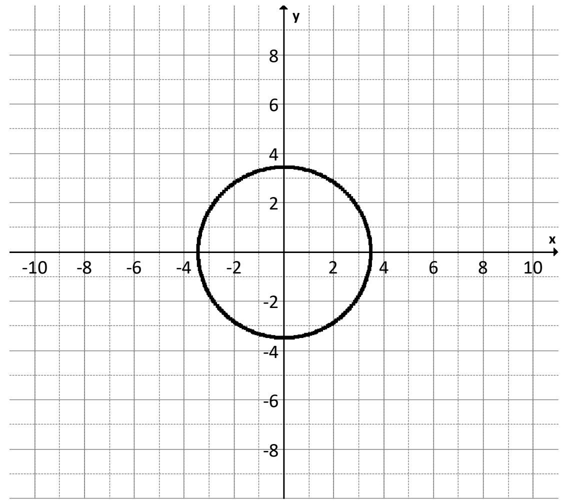
$$2x^2 + 2y^2 - 4x + 16y + 2 = 0$$

Conic Section:



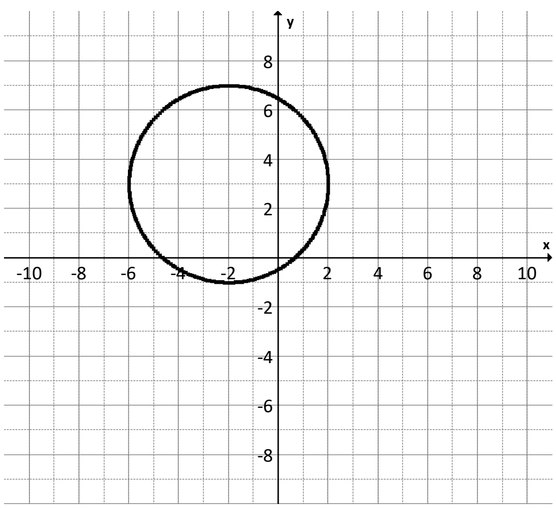
3. Equation: $x^2 + 16y^2 - 64y = 0$

Conic Section:



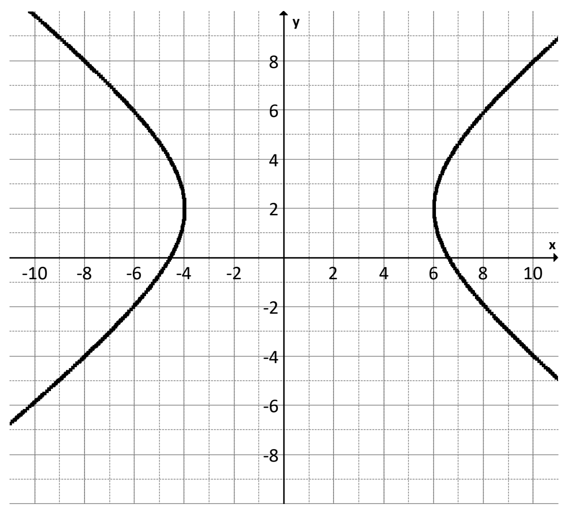
4. Equation: $3x^2 + 3y^2 - 36 = 0$

Conic Section:



5. Equation: $x^2 + y^2 + 4x - 6y - 3 = 0$

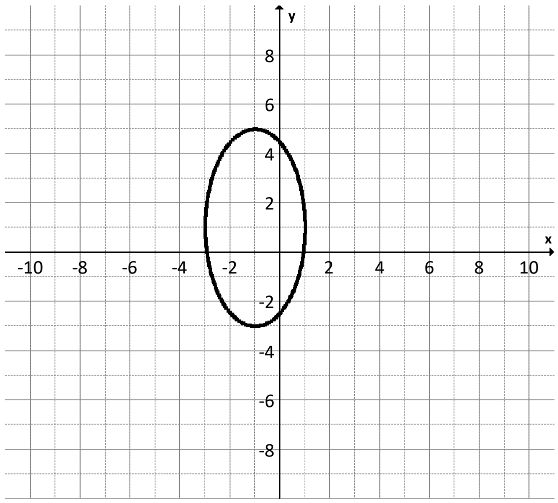
Conic Section:



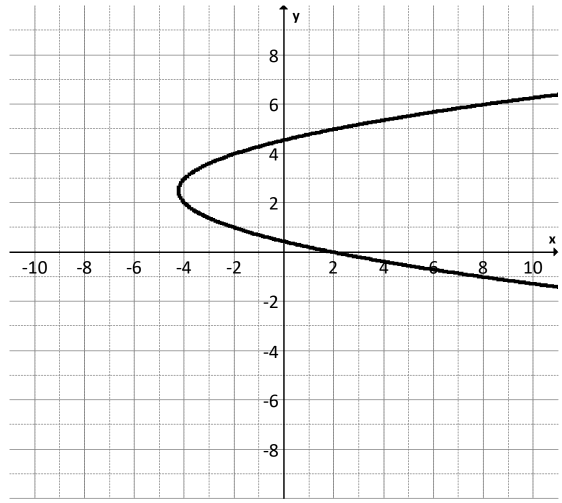
6. Equation:

$16x^2 - 25y^2 - 32x + 100y - 484 = 0$

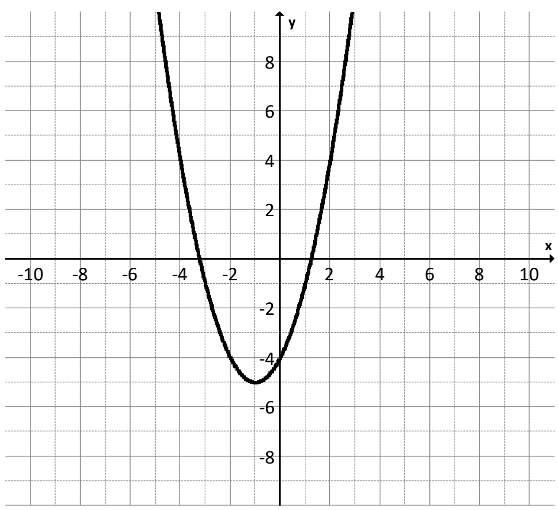
Conic Section:



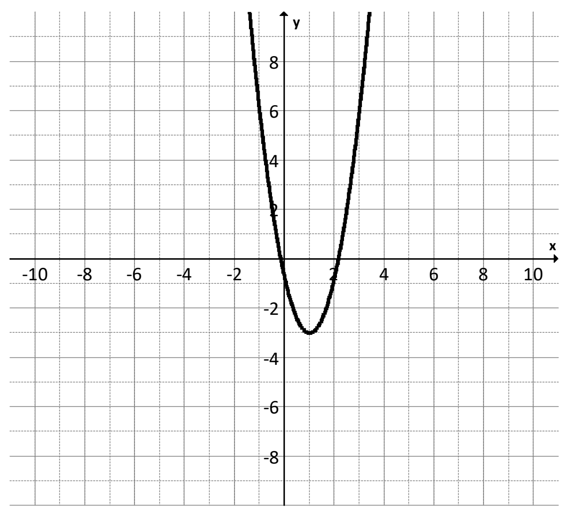
7. Equation:
$16x^2 + 4y^2 + 32x - 8y - 44 = 0$
Conic Section:



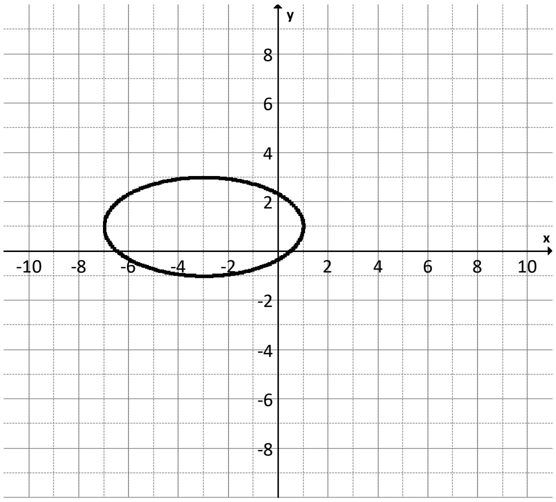
8. Equation: $y^2 - x - 10y + 2 = 0$
Conic Section:



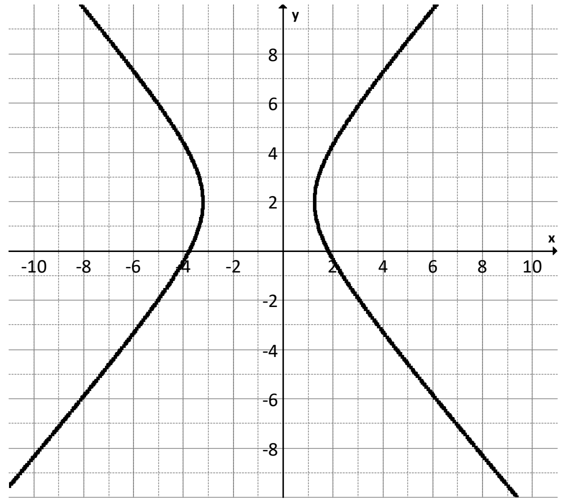
9. Equation: $x^2 + 2x - y - 4 = 0$
Conic Section:



10. Equation: $9x^2 - 18x - 4y - 3 = 0$
Conic Section:




11. Equation: $x^2+4y^2+6x-8y-3=0$
Conic Section:




12. Equation
$7x^2-5y^2+14x+20y-48=0$
Conic Section:

| Circle | Parabola |
|---|---|
| | |
| **Ellipse** | **Hyperbola** |
| | |

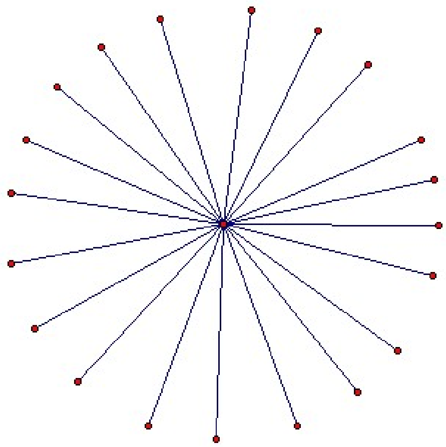
**Practice**

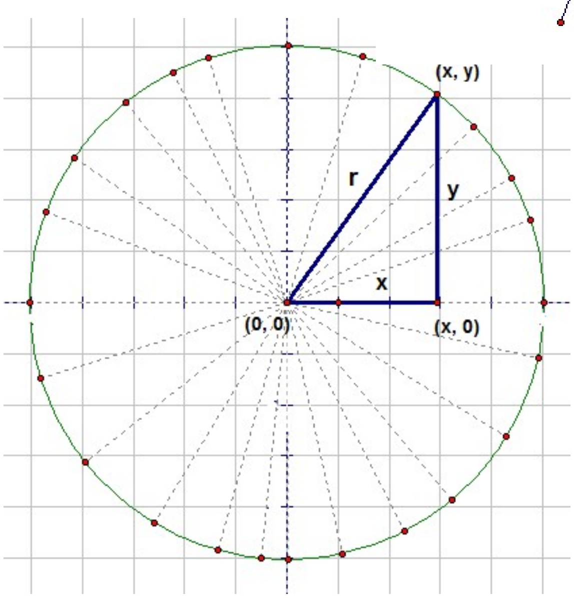
Name the conic from the general equation written below.

1. $4x^2+9y^2=36$
2. $9x^2-4y^2+18x+16y-43=0$
3. $y^2-4x-4y+16=0$
4. $4x^2+y^2+64x-12y+288=0$
5. $x^2+y^2+10x-6y+18=0$
6. $x^2-4y^2-2x-8y-7=0$

**Circles**

By definition a circle is the set of all points on a plane equidistant (radius) from a given point (center).






If this circle is drawn on an axis system, with the center located at (0, 0) with radius r, it is possible to write an algebraic equation for the circle.

Suppose the center point is located at the origin (0, 0). Choosing one point (x, y) allows us to form a right triangle with radius r as the hypotenuse. One leg is the perpendicular segment from (x, y) to the x-axis at point (x, 0). The second leg is the segment from the point (x, 0) back to the origin. Use the Pythagorean Theorem to write an equation for r.

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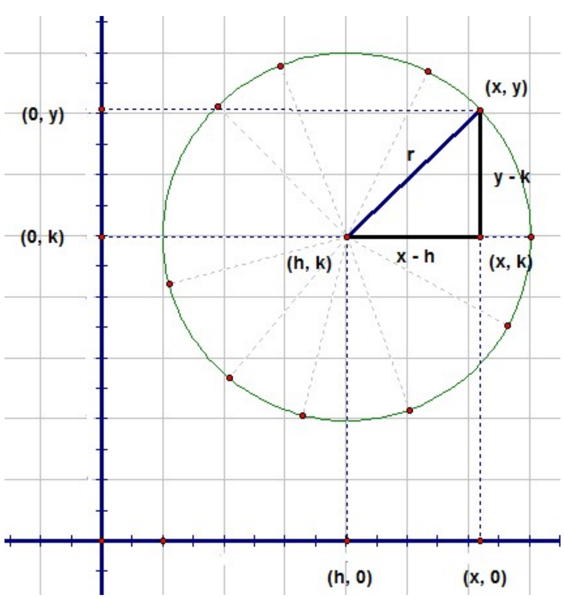


Because this equation is true for every point on the circle, it can be given as the equation of the circle itself.

Now suppose the center point is located away from the origin at point (h, k). Following the procedure used with a circle located with its center at the origin, pick a point (x, y) on the circle and form a right triangle with the other vertices at (x, k) and (h, k). The hypotenuse is r and the legs are $y - k$ and $x - h$. According to the Pythagorean Theorem,

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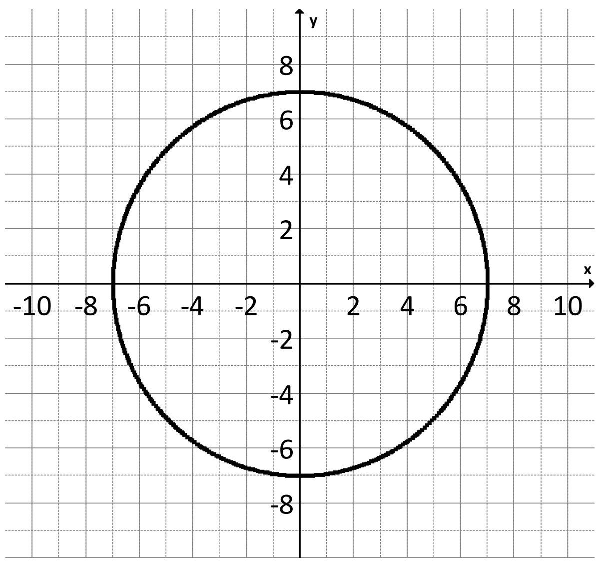
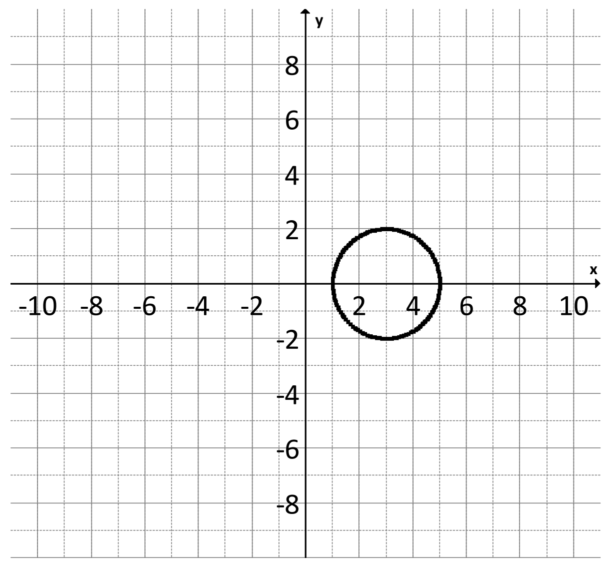
____________________________________ is called the standard form equation for a circle with a center at (h, k) and radius r. By multiplying and collecting terms, the standard form equation can be written as $x^2 + y^2 - 2\mathrm{hx} - 2\mathrm{ky} + h^2 + k^2 = r^2$ which is fits into the general conic equation $\mathrm{A}x^2 + \mathrm{C}y^2 + \mathrm{Dx} + \mathrm{Ey} + \mathrm{F} = 0$. In order to be a circle A = C in the general conic equation.

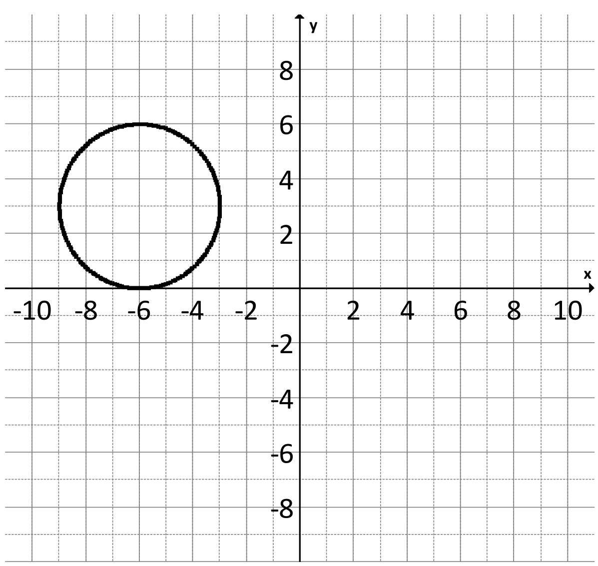
**Using a Circle's Equation**

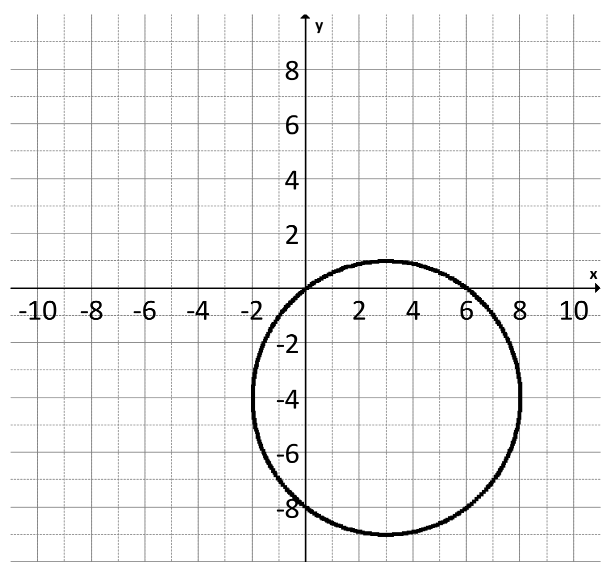
Write the equation of each circle below.




Equation: ____________________ Equation: ____________________

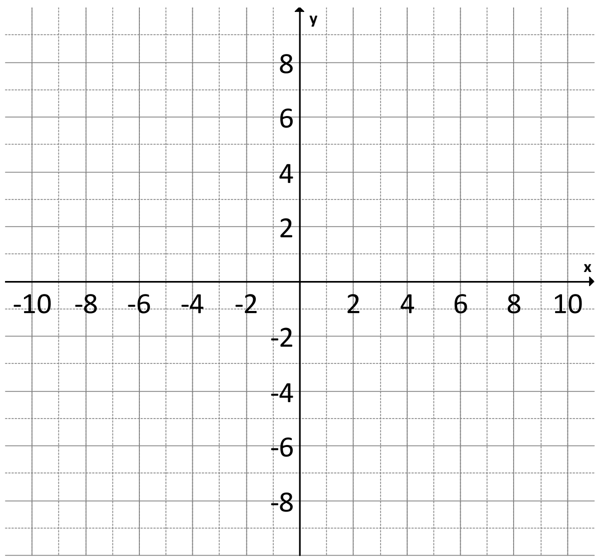


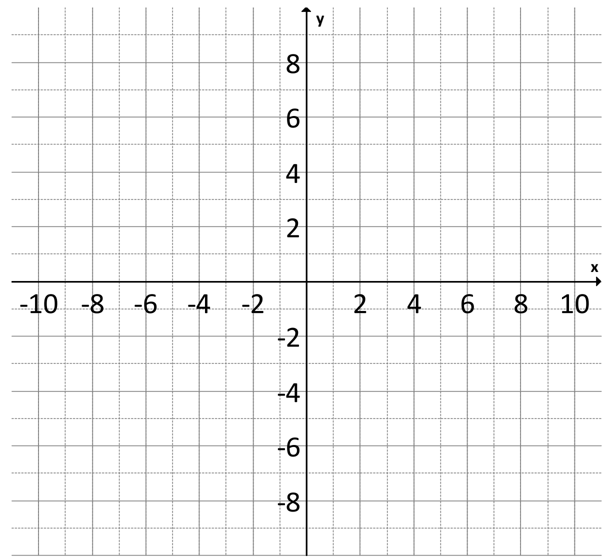

Equation: ____________________ Equation: ____________________

Use the equation to graph each circle.

$(x-3)^2 + y^2 = 16$




$(x+1)^2 + (y+5)^2 = 1$




Now use what you have seen to rewrite the following equations in standard form.

1. $x^2 + y^2 - 8x - 65 = 0$

Equation: ____________________

Radius: _____ Center: _________

2. $x^2 + y^2 + 14y + 24 = 0$

Equation: ____________________

Radius: _____ Center: _________

3. $x^2 + y^2 + 2x - 10y + 22 = 0$

Equation: ____________________

Radius: _____ Center: _________

# Examples

Rewrite in Standard form for a circle: $(x-h)^2+(y-k)^2=r^2$

1. $x^2+y^2+8x=84$

2. $x^2+y^2-18y+65=0$

3. $x^2+y^2+20x-26y+268=0$

4. $x^2+y^2-22y+14x+134=0$

5. $x^2+6x+y^2-4y=12$

6. $2x^2-12x+2y^2+12y=4$

**Completing the Square Practice**

**Write each circle in Standard Form by completing the square.**

1. $x^2 + y^2 + 6x - 6y - 31 = 0$

2. $x^2 + y^2 + 2x + 10y - 10 = 0$

3. $x^2 + y^2 + 16x - 14y + 97 = 0$

4. $x^2 + y^2 + 20x + 12y + 120 = 0$

5. $x^2 + y^2 - 22x + 4y + 89 = 0$

6. $x^2 + y^2 - 16x - 6y + 48 = 0$

7. $2x^2 + 2y^2 - 32x - 28y - 24 = 0$

8. $3x^2 + 3y^2 + 36x - 42y + 30 = 0$

**Answers to Completing the Square Practice**

1. $(x+3)^2 + (y-3)^2 = 49$
2. $(x+1)^2 + (y+5)^2 = 36$
3. $(x+8)^2 + (y-7)^2 = 16$
4. $(x+10)^2 + (y+6)^2 = 16$
5. $(x-11)^2 + (y+2)^2 = 36$
6. $(x-8)^2 + (y-3)^2 = 25$
7. $(x-8)^2 + (y-7)^2 = 125$
8. $(x+6)^2 + (y-7)^2 = 75$

$$x^2 + 20x + y^2 + 16y = -20$$

The equation above defines a circle in the $xy$-plane. What are the coordinates of the center of the circle?

A) $(-20, -16)$

B) $(-10, -8)$

C) $(10, 8)$

D) $(20, 16)$

$$x^2 + (y + 1)^2 = 4$$

The graph of the equation above in the $xy$-plane is a circle. If the center of this circle is translated 1 unit up and the radius is increased by 1, which of the following is an equation of the resulting circle?

A) $x^2 + y^2 = 5$

B) $x^2 + y^2 = 9$

C) $x^2 + (y + 2)^2 = 5$

D) $x^2 + (y + 2)^2 = 9$

$$x^2 + y^2 - 6x + 8y = 144$$

The equation of a circle in the $xy$-plane is shown above. What is the *diameter* of the circle?

$$x^2 + 8x + y^2 - 6y = 24$$

The graph of the equation above in the $xy$-plane is a circle. What is the radius of the circle?

Which of the following is an equation of a circle in the $xy$-plane with center $(0, 4)$ and a radius with endpoint $\left(\frac{4}{3}, 5\right)$?

A) $x^2 + (y - 4)^2 = \frac{25}{9}$

B) $x^2 + (y + 4)^2 = \frac{25}{9}$

C) $x^2 + (y - 4)^2 = \frac{5}{3}$

D) $x^2 + (y + 4)^2 = \frac{3}{5}$